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AN

ANALYSIS

OF THE PRINCIPLES

OF

NATURAL PHILOSOPHY.

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NEMO *expectet magnum progressum in scientiis, praesertim in parte earum operativa, nisi Philosophia Naturalis ad scientias particulares producta fuerit, et scientiae particulares rursus ad Naturalem Philosophiam reductae. Hinc enim fit, ut Astronomia, Optica, Musica, et plurimae artes Mechanicae, nil fere habeant altitudinis in profundo; sed per superficiem et varietatem rerum tantum labantur: quia postquam particulares istae scientiae dispertitae et constitutae fuerint, a Philosophia Naturali non amplius aluntur, quae ex fontibus et veris contemplationibus motuum, radiorum, sonorum, novas vires et augmenta illis impertiri potuerat.*

BACON Nov. Org. Aph. lxxx.

AN

ANALYSIS, &c.

LECTURE I.

I. THE bufinefs of Natural Philofophy is to defcribe the phænomena of the univerfe; to trace the relations and dependencies of caufes; and to make art and nature fubfervient to the neceffities of life.

II. It's reality depends on the reality of our fenfations; and is not therefore affected by the exiftence or non-exiftence of an external, material world.

III. Natural Philofophy has originated from the wants and defires of man.

IV. The inducements to purfue the ftudy of this fcience are, 1ft. It's extenfive influence in improving arts and manufactures. 2dly. It's lay-

ing a ſure foundation of natural religion. 3dly. It's promoting the proper diſcharge of our duty in it's three great branches, to God, to man, and to ourſelves: to God, in acquiring proper ſentiments of his attributes, by contemplating his works; to man, in cultivating the valuable and neceſſary arts of life, by a ſkilful application of the powers of nature; to ourſelves, by exerciſing our minds on the moſt glorious and intereſting objects; and thus ſtrengthening our reaſon, ſhaking off contracted prejudices, and every day making a farther progreſs in the contemplation of truth.

V. The difference between the Epicureans and Fataliſts conſiſts in this, that the one ſect maintains the world to be a work of mere chance, the other of abſolute neceſſity.

VI. Both theſe Atheiſtical ſects are refuted, the one by the uniformity, the other by the variety of nature.

Wolfe and Leibnitz have introduced the doctrine of neceſſity under a different form from that of the ancient Fataliſts. The neceſſity of the Ancients was a blind neceſſity, deſtitute of all wiſdom and choice; the neceſſity of Leibnitz conſiſts in this, that an intelligent being muſt be determined by *a ſufficient reaſon*; for nothing happens without a ſufficient reaſon, why it ſhould be ſo rather than otherwiſe; but that ſufficient reaſon according to him lies in the differences of things and motives, ſo that the mind is neceſſarily determined

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mined in its volitions and elections by the greateſt apparent good; and it is impoſſible to make a choice between things perfectly alike; from whence he infers, that two things perfectly alike, differing *ſolo numero*, or only becauſe they are two, could not have been produced by the Deity. But to this doctrine of Philoſophical Neceſſity it is replied, that the Will is a faculty, which is phyſically indifferent to acting or to not acting, or to acting in any particular manner, notwithſtanding the different affections or paſſions of the mind, raiſed by different objects; and which, by merely chooſing a thing, can make it agreeable, though it had no agreement with any natural appetite, nay were contrary to them all; and for the Will to chooſe a thing in order to pleaſe itſelf in the choice, is no more to chooſe without reaſon, than to build a houſe, in order to preſerve one from the inclemency of the weather, is to act without reaſon. Neither can the agent in ſuch a caſe be ſaid to be determined by chance, becauſe, here is no room for chance, if by chance be underſtood that which happens beſide the intention of the agent; for the very choice is the intention of the agent, and it is impoſſible that an agent ſhould intend beſide his intention. See King's Origin of Evil, and Biſhop Law's Notes; and alſo Limborch Theol. Lib. 2. cap. 23.

VII. Syſtem conſiſts in the claſſification of phænomena.

VIII. The obſtacles which impeded the Ancients in founding a rational ſyſtem of Phyſics were, Firſt, The want of many inſtruments diſcovered by the moderns. Secondly, Their not having had recourſe

to Mathematical reaſoning. Thirdly, The influence of the Ariſtotelic Philoſophy.

IX. Lord Verulam was the great founder of Experimental Philoſophy.

To render Natural Philoſophy truly fruitful, he propoſed that two different modes of reaſoning ſhould be duly combined, which he called the *Scala Aſcenſoria* & *Deſcenſoria*, the former leading from experiments to general concluſions, the latter from general concluſions to new diſcoveries. Theſe are otherwiſe called the Analytic and Synthetic methods, which were afterwards purſued by Newton with ſo much ſucceſs.

X. To ſolve a phænomenon is only to deduce it from, or ſhew that it is included under ſome general head; not to aſſign its efficient cauſe, which perhaps may be unmechanical.

XI. Occult cauſes are ſuch as have not been proved to exiſt by any experiment or concluſive chain of reaſoning.

XII. Magnetiſm, Electricity, or Gravity are not to be rejected as occult cauſes, becauſe their nature is unknown.

XIII. Analyſis ſhould precede Syntheſis, otherwiſe we can never be certain, that we do not aſſume powers which have no exiſtence in nature.

XIV. In the Newtonian method of Philoſophizing three things are to be conſidered; 1ſt. The topics

pics of experimental enquiry and obfervation. 2dly. The manner of inftituting experiments. 3dly. The rules to be obferved in reafoning on experiments, and deducing general conclufions.

XV. Inftances of the topics of experimental enquiry are various, as

1. The progrefs of nature in generating or deftroying, encreafing or diminifhing the fame property in the fame body,

Thus water when cold to a certain degree becomes folid and confiftent; when it fuffers only a gentle heat, it is liquid; when heated to the 212th degree of Farenheit, it is converted into an elaftic vapour.

2. The procefs of nature in diftributing the fame property to different fubftances.

Thus the refractive and difperfive powers are different in different kinds of glafs,

3. The different properties with which the fame property is combined in different bodies.

Thus folubility in water is a property common to every fpecies of faline fubftance.

4. What thofe properties are which are feldom or never found united.

As malleability and tranfparency.

5. The tranfition of nature from one quality to another.

As, in fermentation from fweetnefs to acidity, and from acidity to alkaline faltnefs.

6. Whether

6. Whether the ſame effect in different bodies be produced in the ſame or different ways.

As whether the emiſſion of light is produced in the ſame way or not by the Sun, culinary fire, electricity, putrefaction.

7. In what caſes the ſame cauſe produces different, and ſometimes contrary effects.

Thus cold generally produces contraction, but ſometimes dilatation.

8. To determine whether a known effect be produced by one or more cauſes; and if by more than one, what is the particular influence of each.

As what is the peculiar influence of each ingredient in the fulmination of gun-powder.

9. The ſimilitude or diſſimilitude of analogous parts or operations, in different individuals or ſpecies.

As in comparing the ſpine of man and four-footed creatures.

10. How the uſual operations of nature are controlled and counteracted.

As how a fire ceaſes to burn by being expoſed to the Sun's rays.

11. Thoſe productions which differ from the reſt of the ſame ſpecies.

As the unuſual refraction of Iceland Cryſtal.

12. Thoſe productions of nature which are imperfect.

As the various imperfect cryſtallizations of Baſalt.

13. The

13. The properties of things in their higheſt and loweſt ſtate of perfection.

As of the beſt and worſt conſtructed ſhip.

14. A compariſon of the works of nature and art.

As of the valves of a pump with the valves of the heart and veins.

XVI. The manner of inſtituting experiments, as laid down by Bacon, is

1. Variation of the experiment, which is three-fold,

1ſt. Of the ſubject.

As whether fixed air will ſtop the putrefaction of the living body, in the ſame manner as in the fleſh of a dead animal.

2dly. Of the efficient cauſe.

As whether a vegetable growing in a veſſel of confined air, will render that air noxious, in the ſame manner as an animal living in it.

3dly. Of quantity.

As whether works executed *in the great*, are equally perfect with their ſmaller models.

2, Production of the experiment, which is two-fold, repetition and extenſion: as where an experiment is either repeated, or puſhed on to a greater degree of ſubtilty.

3. Tranſlation of the experiment, which is three-fold, either from nature or chance to art; or from one

one art to another; or from one branch to another of the fame art.

4. Inverfion of the experiment, or where we try the contrary of that which has been difcovered by experiment.

5. Compulfion of the experiment, or where an experiment is urged on fo far, as that a fubftance is actually deprived of fome quality, or the effect of fome quality counteracted.

6. Application of the experiment, or the fortunate transferring of an experiment to fome other ufeful experiment.

7. Conjunction of the experiment, as when certain things have certain properties when feparate, it is tried whether they have the fame or different powers when conjoined.

8. The chances of experiment, or the trying a conclufion for no other reafon than becaufe it never was tried before.

XVII. The rules to be obferved in reafoning on experiments and deducing general conclufions from particular inftances, are as follow:

Rule 1. More caufes of natural things are not to be admitted, than are both true and fufficient to explain the phænomena.

Caufes are either experimental or rational: experiment is the only ftandard of experimental caufes; perception of the

the neceſſary connexion of events is the ſtandard of rational cauſes. See R. Young on the Mechaniſm of Nature.

Rule 2. Of natural effects of the ſame kind, the ſame cauſe is to be aſſigned.

Rule 3. The qualities of all bodies which cannot be increaſed or diminiſhed, and that agree to all bodies on which experiments can be made, are to be reckoned qualities of all bodies whatſoever.

Rule 4. Propoſitions collected from phænomena by induction are to be deemed, notwithſtanding contrary hypotheſes, either accurately true, or very nearly ſo, until other phænomena occur, by which they may be rendered more accurate, or overturned.

This laſt rule is neceſſary in order to prevent the progreſs in phyſics from being impeded by hypotheſes.

XVIII. Hypotheſes may be admitted in Philoſophy, under the following reſtrictions: 1st. That they be not arbitrary, but ſupported by the reaſon and analogy of things. 2dly. That they ſolve all the phænomena to which they relate.

XIX. Under theſe regulations hypotheſes are uſeful. 1st. They help the memory, and 2dly. They lead to new diſcoveries.

LECTURE II.

1. NATURAL Philoſophy is the ſtudy of natural phænomena, in order to diſcover the forces which produce them; and from thoſe forces, aſſumed as cauſes, to explain other phænomena.

2. It is divided into two branches, 1ſt. That which treats of the actions of bodies, by which their eſſential properties are changed. 2dly. That in which thoſe properties are not changed, which is Mechanical Philoſophy.

3. Mechanical Philoſophy is twofold, Terreſtrial Mechanics and Phyſical Aſtronomy.

4. The object of Natural Philoſophy is matter; by which we are to underſtand an aſſemblage of the ſenſible qualities of extenſion, figure, ſolidity, inactivity, and mobility, together with a power of exciting certain ſenſations in us, and of changing the qualities of other bodies.

5. Ariſtotle

5. Aristotle defined matter by negatives, as having neither quantity, nor quality, nor essence.

6. Des Cartes made the essence of matter to consist in extension; but this doctrine is refuted by Locke.

7. The essence of matter consists in extension, impenetrability and inertness.

Though we know in what the essence of matter *in general* consists, yet we are entirely ignorant of the essence of *individual beings*.

8. All bodies are extended.

9. All bodies have some determinate figure.

10. All bodies are divisible either in fact, or in imagination.

If we speak of that division which we conceive in our imagination, body is infinitely divisible.

11. By art body may be divided into parts of surprising minuteness.

12. But Nature affords corpuscles of such extreme minuteness, as surpasses all imagination.

13. Matter is demonstrated to be infinitely divisible in a mathematical view.

14. There is a limit however to the actual division of matter, which though different perhaps in different bodies, art cannot exceed, and nature in her operations never appears to transgress.

15. Matter therefore, we are to ſuppoſe, was originally created in primary particles of definite magnitude, whoſe parts are imaginary only, not real.

Theſe elementary particles therefore are totally different from the Monads of Wolfe and Leibnitz, which were the ultimate and ſmalleſt particles of bodies, deſtitute of magnitude, and incapable of any farther diviſion, even in imagination. But if theſe monads be extended, they muſt be capable of ſubdiviſion, at leaſt in imagination; and if they be not extended, they cannot be the elements of extended body.

Hence is ſhewn the inconcluſiveneſs of Prieſtley's Hypotheſis, who ſuppoſes, that if the forces of attraction and repulſion were to ceaſe, matter would at the ſame time be actually annihilated.

16. All bodies are porous.

17. From the porouſneſs of bodies and the extreme minuteneſs of their conſtituent particles, it happens, that fluids will inſinuate themſelves into all bodies; and that ſometimes a mixture of two fluids will be leſs in bulk, than when they are ſeparate.

18. From the porouſneſs of bodies it follows, that the ſame quantity of matter is not always contained under the ſame magnitude: and hence ariſes the idea of Denſity, or that property of bodies by which they contain a given quantity of matter in a given bulk; which therefore is directly

as

as the quantity of matter and inverſely as the magnitude of the body.

Since from experiments on pendulums it appears, that the quantity of matter in different bodies is proportional to their weight, it follows, that the denſity of any body is directly as its weight, and inverſely as its magnitude.

19. From the extreme tenuity to which matter may be reduced, the following paradoxes are derived.

1ſt. A hollow cube whoſe ſide, and therefore alſo a concave ſphere whoſe radius ſhall be equal to any line however great, may be formed from any quantity of matter however ſmall.

2dly. It is poſſible, that any given quantity of matter, however ſmall, may be ſo diffuſed through any given finite ſpace, however great, and ſo fill it, that there ſhall be no pore in it, whoſe diameter will exceed the ſmalleſt given line.

3dly. There may be two bodies of equal magnitude, whoſe quantities of matter are unequal in any given ratio, and yet the ſum of their pores ſhall be very nearly equal.

This paradox is true only on the hypotheſis, that the ſolid matter of the denſer of the two bodies occupies but a ſmall part of the entire ſpace, through which the whole matter is diffuſed.

20. The pores of animals are not to be confounded with the inorganic pores of dead matter, which,

which, not belonging to any veſſels, are diffuſed through the entire maſs; but the pores of animals are the orifices of the ſyſtem of veſſels called Abſorbents or Exhalants.

21. Space is the order of things which coexiſt.

It is greater or leſs according to the number of like things that either are, or may be interpoſed between its extremes. It is not therefore any thing really exiſting; but is a mere abſtract idea, ariſing from our idea of the actual or poſſible ſituation of things amongſt themſelves.

22. Space is twofold, Abſolute and Relative.

Abſolute Space, or ſpace in the abſtract, is the order of things which do not, but may exiſt together: Relative Space is the order of things actually ſo exiſting; and therefore is the ſame with Extenſion. Abſolute Space conſidered in one direction only is Diſtance; in two, is Mathematical Surface; in three, is Mathematical Body. Relative Space in one direction is Length or Breadth; in two, is Phyſical Surface; in three, is Extenſion.

23. Place is the relation of diſtance between any thing and any two or more points, which being conſidered at reſt, keep the ſame diſtance one from the other.

If the whole Univerſe be ſuppoſed to be finite, we can conceive it moved forward *in directum;* becauſe we can conceive other external bodies, not part of the Univerſe, with reſpect to which that finite ſyſtem, called the Univerſe, may change its relation of diſtance. But if the Univerſe

verſe be ſuppoſed infinite, that is, to compriſe not only all bodies which actually exiſt, but which can be even imagined as poſſibly exiſting, we cannot conceive ſuch rectilineal motion.

24. Mobility is that property of body, by which it is capable of exiſting in different parts of ſpace.

25. Solidity is that property, by which a body excludes all others from the place it occupies, 'till it has left it.

LECTURE

LECTURE III.

1. MOTION being a simple idea cannot be defined; we therefore describe it only by its sensible effects, when we say, that it is a continual and successive change of place.

2. Change of place is not motion, 1st. Because the subject of motion is not the same with the subject of the change of place. 2dly. Tho' the changes of place of a body be essentially different, yet its motion may be the same. 3dly. Motion is the action of a body, by which it changes place.

3. Aristotle's definition of motion, viz. "The energy of what exists in power, considered as so existing," signifies, "The actual exercise of the capacity which a being has of becoming an agent, considered as rendering that being an agent in fact, which before had simply the power of being so."

4. Aristotle under this genus of motion comprehended six different species, viz. 1st. Transition

tion or change of place. 2nd Aliation or change of quality. 3rd. Augmentation. 4th. Diminution. 5th. Corruption. 6th. Generation.

5. Hence Tranſition, or Local Motion, is defined, The act or energy of a being which has the capacity of changing place, conſidered as having that capacity.

6. It has been diſputed that motion, as defined by Philoſophers, is impoſſible.

Let us ſuppoſe, ſays Gianvil, the circumference of a wheel to be divided according to the alphabet; ſince in motion there is change of place, and in the motion of a wheel there is a ſucceſſion of one part to another, in the ſame place; it ſeems inconceivable that *A* ſhould move, until *B* has left its place; now it cannot move, but it muſt acquire ſome place or other: and it can acquire none but *B*'s, which we ſuppoſe to be moſt immediate to it; but the ſame place cannot contain them both; and therefore *B* muſt leave its place, before *A* can get into it; but *B* cannot move except into the place of *C*; and *C* muſt leave that place, before *B* can come into it; ſo that the motion of *C* will be pre-required likewiſe to the motion of *A*; and ſo onward to *Z*, which is next to it; for the ſame reaſon *Z* cannot move, until *A* moves; neither will *A* be able to move, until *Z* hath, (as has been ſhewn already) ſo that the motion of every part will be pre-required to itſelf, which is abſurd. Scepſis Scientifica, page 4. This objection is eaſily anſwered by aſſerting, that the parts do not move in ſucceſſion,

but all at once. To this Glanvil anſwers, that we cannot conceive in a ſucceſſion, but that ſomething ſhould be firſt, and that motion ſhould begin ſomewhere. This is admitted; for the motion of each part, ſeparately conſidered, begins from the point, at which it was at the beginning of the motion of the wheel, ſo that every part commences its motion from a different point; but all the motions commence at the ſame inſtant. When it is aſſerted, that the points do not move in ſucceſſion, that ſucceſſion relates to the order of time, not to the points of ſpace.

It is farther objected againſt the reality of motion, that it infers an equality between the circumferences of unequal circles; for if there be two wheels of unequal diameters, fixed on the ſame axle, and the larger revolve on a plane; in the ſame time that it deſcribes, on that plane, a ſpace equal to its circumference, the ſmaller wheel likewiſe, if it be ſuppoſed to be touched by another plane, will, in one revolution, deſcribe the ſame ſpace.

In anſwer to this difficulty, which was firſt remarked by Ariſtotle, we are to obſerve, that the circular motion of the wheel is cauſed by the reſiſtance of the plane, on which it moves. Now this reſiſtance is equal to the force with which the wheel is drawn forward in a right line, ſince it deſtroys the motion which the point of the wheel, that touches the plane, ought to have in that direction. The cauſes of theſe two motions, one direct, the other circular, are therefore equal, and conſequently their effects; hence the wheel deſcribes, on the plane, a right line equal to the circumference. With reſpect to the leſs wheel on the ſame axis, it is drawn directly

directly forward, with the ſame force with which the large wheel is drawn; but ſince the circumferences of the two wheels revolve in the ſame time, the circular motion of any point in the periphery of the leſs wheel, muſt be leſs than that of the greater; therefore ſince this point neceſſarily deſcribes a right line equal to the circumference of the greater wheel, it follows, that it muſt do ſo partly by ſliding forward, and partly by revolving. If the ſmaller wheel were to revolve on the plane, and carry the larger with it, which at the ſame time ſhould be touched by an imaginary plane, the larger wheel would deſcribe, on that imaginary plane, a right line equal to the circumference of the leſs wheel, partly by revolving, and partly by ſliding backward.

7. Motion is twofold, Abſolute and Relative.

The motion of a body is Abſolute, when the immediate cauſe of the change of its diſtance, with reſpect to other bodies, is in the body: it is Relative, when the cauſe of the change is in other bodies.

8. Abſolute and relative motions are diſtinguiſhed by their cauſes and effects.

The cauſes by which they are diſtinguiſhed are the forces impreſſed on the bodies to generate motion. Now 1ſt. Abſolute motion is neither generated nor altered except by forces impreſſed on the body moved; but relative motion may be generated or altered without any forces impreſſed on the body; for it is ſufficient that ſome force be impreſſed on other bodies, to which it is referred, that by their giving way the relation may be changed. 2dly. Abſolute motion always ſuffers ſome change from forces impreſſed on the body moved; but relative motion is not neceſſarily changed by

ſuch forces. The effects by which Abſolute and Relative motions are diſtinguiſhed from each other, are the forces of receding from the axis of motion.

9. Time is the order of things which exiſt in ſucceſſion.

It is greater or leſs according to the number of like ſtates interpoſed between any two given things or ſtates thus ſucceſſively exiſting. It is not therefore any thing really exiſting, but is a mere abſtract idea of relation.

10. Time is twofold, Abſolute and Relative.

Abſolute time, or time in the abſtract, is the order of things which do not ſucceed, nor have actually ſucceeded each other, but which may be conceived as poſſibly ſo ſucceeding: Relative time is the order of things actually ſucceeding each other; and this is what we call Duration. Hence we may obſerve, that duration is to time, what extenſion is to ſpace; for as extenſion is ſpace within which co-exiſting things are actually interpoſed; ſo duration is time within which ſucceſſive ſtates have been actually interpoſed.

11. Any finite ſpace is deſcribed in time.

12. Velocity is that affection of the motion of a body, by which it deſcribes a certain ſpace in a certain time.

13. Thoſe quantities alone are naturally meaſurable which conſiſt of parts.

An arbitrary meaſure is aſſigned to quantities which are eſtimated by degrees, by referring them to ſome meaſurable

quantity

quantity to which they are related; thus the meaſure of velocity is the ſpace deſcribed by a body in a given time.

14. When a body moves with a uniform velocity, the ſpace deſcribed is proportional to the time of its motion.

15. When bodies move with different uniform velocities, the ſpaces deſcribed are proportional to the times and velocities jointly.

Let V and v be the velocities of two bodies A and B; T, t the times of their motion; S and s the ſpaces deſcribed; alſo let $\grave{S}$ be the ſpace deſcribed by B in the time T; then $S : \grave{S} :: V : v$, and $\grave{S} : s :: T : t$; therefore $S : s :: TV : tv$. See Wood's Mechanics.

16. If the motion of a body be accelerated or retarded, the meaſure of the velocity at any point, is the ſpace which would have been deſcribed, in a given time, if the motion had continued uniform from that point.

17. The meaſure of motion is the product of the quantity of matter and velocity.

18. Whatever changes, or tends to change the ſtate of reſt, or uniform rectilineal motion of a body is called force.

Thus preſſure, impact, gravity, electricity, magnetiſm, are forces. When a force produces its effect inſtantaneouſly, it is ſaid to be impulſive; when it acts inceſſantly, it is conſtant.

Conſtant

Conſtant forces are of two kinds, uniform and variable: an uniform force always produces equal effects, a variable force unequal effects, in equal ſucceſſive portions of time.

19. Forces are meaſured by the effects, which they produce under the ſame circumſtances.

Thus impulſive forces are meaſured by the whole effects produced; uniform forces, by the effects produced in equal times; and variable forces, by the effects which would be produced in equal times, were they to become and continue uniform for thoſe times.

20. The effects produced by the actions of forces are of two kinds, velocity and motion.

Force generating velocity is called the accelerating force; generating motion, the moving force.

21. The accelerating force is meaſured by the velocity uniformly generated in a given time, without any regard to the quantity of matter moved.

22. The moving force is meaſured by the quantity of motion uniformly generated in a given time.

23. The moving force varies as the accelerating force and the quantity of matter jointly.

24. The accelerating force varies as the moving force directly, and the quantity of matter inverſely.

25. The general laws of motion are three: 1st, Every body perſeveres in its ſtate of reſt, or uniform

form motion in a right line, until a change is effected by ſome external cauſe. 2dly. Every motion and change of motion is proportional to, and in the direction of the force impreſſed. 3dly. Action and Reaction are equal and contrary.

26. The laws of motion are proved by experiments, 1ſt. which relate directly and immediately to the laws themſelves; or 2dly. which reſpect the conſequences, which would reſult from theſe laws, were they true.

27. Though all motions which fall under the cogniſance of our ſenſes gradually decay, yet this does not affect the truth of the firſt law, becauſe we can, in all caſes, aſſign the external cauſes of that decay.

28. The Second law of motion is not an identical propoſition; 1ſt. Becauſe the direction of the force impreſſed is the right line, in which the impelling body is moving at the inſtant of impact; or in which the preſſing body would move, were there no obſtacle to prevent it. 2dly. Becauſe the proportion of two forces may, in many caſes, be aſcertained, independent of the motion produced by them.

In comparing the changes produced in the motion of bodies according to the ſecond law, the times of effecting ſuch changes are ſuppoſed equal.

29. In eſtimating the effect of any force, we are to conſider, 1ſt. Whether the whole force, or only a part of it be really efficient. 2dly. The direction in which the force acts.

30. The Third law of motion ſignifies, that there are equal changes made in the motions and ſtate of bodies by their mutual actions, and in contrary directions.

31. The Firſt law of motion has reſpect to the continuance of motion in bodies, without any alteration of the motion, except ſo far as ſubſequent cauſes operate; the Second aſſigns the quantity and nature of ſuch alterations; and the Third has regard to the mutual circumſtances of the patient which ſuffers alteration from any cauſe, and of the agent producing the alteration.

32. Since the changes made in the motions of any two bodies which act upon each other are equal and in contrary directions, it follows that the quantity of motion, eſtimated in the ſame direction, is not changed by the mutual action of the bodies.

33. The laws of motion relate immediately to the actions of bodies in free ſpace.

When bodies move on fixed axes, the energy of the moving force and reſiſtance of the body moved will depend on the diſtance from the axis; in theſe caſes, the inertia of

of the parts of the fyftem, by which they refift motion, muft firft be calculated, and a mafs, whofe inertia is equivalent to it, muft be fubftituted for the given fyftem; and the quantity of the moving force, when moving with the fame velocity with the matter moved, is likewife to be computed. The moving force and mafs moved being thus afcertained, the refulting motions may be calculated; and it appears, that in rotatory motions on fixed axes, or on moveable axes in free fpace, as well as in direct rectilineal motions, action is always equal to reaction, and the quantity of motion permanent.

LECTURE IV.

1. THE inertia of bodies is that property, by which they continue in their ftate of reft or uniform rectilineal motion, until acted on by fome external caufe.

This property is improperly called a force. 1ft. Becaufe were it actually fuch, it muft be of fome definite quantity in a given body; and therefore an impreffed force lefs than that would not move the body; whereas any impreffed force, however fmall, whether impulfive or conftant, will move any body however great. 2dly. It is improper, becaufe it feems to indicate an active power refident in matter.

2. Bodies by their inertia, when at reft, refift the impulfe of any force tending to fet them in motion; and when in motion, refift the impulfe of any force tending to augment or diminifh their motion.

Since

Since in the same given circumstances, equal quantities of matter are found to resist equally, we say that the inertia is proportional to the quantity of matter.

3. To estimate the motion of bodies, their inertia must be considered; whereas the equilibrium produced by the action of any forces depends only on the quantities and directions of the forces, without any regard to inertia.

4. The continuation of motion does not require constant action.

1st. While a pendulum descends from rest, two forces act upon it, gravity and the reaction of the point of suspension; after the body has attained the lowest point, we find that it ascends nearly to the same height, from which it fell; but this cannot be effected by the two forces, which act on it after it has left the lowest point, for they would cause a descent of the body; it ascends therefore in consequence of the forces, which had continued to act upon it during its descent; and which, from the lowest point, not only cease to urge the body forward, but even endeavour to move it backwards. 2dly. Experiment shews that continued action produces accelerated motion; therefore uniform motion must be the consequence of a force which is not constant, that is, of a force impressed, which has ceased to act.

5 The resistance of matter does not depend solely on its gravity.

For if two bodies of equal weight, be connected by a ſtring which paſſes over the groove of a wheel, whoſe axle turns on friction wheels, they may be conſidered as deſtitute of gravity with reſpect to any force tending to move them in a vertical direction; yet it is found, by experiment, that, in this caſe, they do reſiſt; and that the reſiſtance is proportional to the quantity of matter.

6. The three laws of motion are reducible to the inertneſs of matter.

This inertia however is not a principle antecedently known, from which thoſe laws may be derived; but we firſt learn the truth of the laws, by experiment; and then find them reducible to this one principle.

7. From the inertneſs of matter, the exiſtence of an interſtitial vacuum may be demonſtrated.

The celeſtial ſpaces muſt be either vacant, or filled with a very attenuated and ſubtile medium; for this medium, whether it be the cauſe of gravity or not, muſt be inert, and therefore diminiſh the motion of any body moving through it; but ſince the motions of the planets are inſenſibly reſiſted, the denſity of the medium muſt be inſenſible; that is, its vacuities muſt be very great with reſpect to its ſolid matter.

8. If a body be acted on by two forces, which would ſeparately cauſe it to deſcribe the adjacent ſides of a parallelogram, with an uniform motion, it will, by their joint action, deſcribe the diagonal in the ſame time, in which it would have deſcribed either

either side, had the impressed forces acted separately: and the motion in the diagonal will be uniform.

This is called the Composition of motion: and in the same manner as the compounded action of two forces may be represented by one right line, which is the diagonal of a parallelogram, so the action of a single force, expressed by one right line, may be resolved into the actions of two other forces, represented by the contiguous sides of a parallelogram, of which the given line is the diagonal.

9. If two sides of a triangle, taken in order, represent the spaces which a body would uniformly describe, in a given time, by two forces acting separately; when these forces act conjointly, the body will describe the third side uniformly, in the same time.

10. If the sides of any polygon taken in order, except the last, represent the space which a body would uniformly describe, in a given time, by as many different forces acting separately, when these forces act conjointly, the body will describe the last side uniformly, in the same time.

11. If a body be acted on by two similar variable forces, whose directions and magnitude are expressed by the adjacent sides of a parallelogram, concurring in the body, it will describe the diagonal of the parallelogram.

Fig. 1. Let the forces act by impulses, at the beginning of equal particles of time, and let *BD*, *DE*, *EF*, and *BG*, *GH*, *HI*

HI be the relative magnitudes of corresponding impulses. By the action of the two first impulses, the body will describe the diagonal *BK*; and by the two next, the diagonal *KL*; but since the forces are similar, the parallelograms *DG*, *NV* are similar, and therefore they exist about the same diagonal *BM*. Let now the particles of time be evanescent, and the forces incessant, and the same demonstration will obtain.

12. If the forces be constant, the velocity in the diagonal will be uniformly accelerated.

13. Every body which moves in a curve line, is acted upon, at least, by two dissimilar forces.

14. If a body be acted on by three forces, represented in quantity and direction by the three sides of a triangle, the body thus acted on will continue at rest.

15. If a body be acted on by any number of forces, represented in quantity and direction by the sides of a polygon, the body thus acted on will continue at rest.

16. If a body be acted on by four forces, in different planes, and remain at rest, those forces are to each other as the three sides and diagonal of a parallelopiped, parallel to their directions respectively.

In general, if a body be acted on by any number of forces in the same or different planes, they may always be reduced to two in the same plane; and consequently a single force may be assigned equipollent to them all.

17. If

17. If three fimilar variable forces, acting on a body, be to each other as the fides of a triangle, parallel or perpendicular to their directions, the body will continue at reft.

18. If the motion of a body, in any direction, be reprefented by a given line, its motion in any other direction will be found, by refolving the given motion into two, one of which fhall be perpendicular, and the other parallel to the direction in which the motion is required.

19. In the refolution of forces, the whole quantity of force is increafed; in compofition, diminifhed.

20. The effects of forces, when eftimated in given directions, are not altered by compofition or refolution.

21. The relative motions of bodies are the differences of their abfolute motions.

22. If the eye of a fpectator move, the apparent motion of a fixed point is the fame as if the eye were fixed, and the motion of the point were equal and contrary to that of the eye.

Fig. 2. Let e, E be any two contiguous pofitions of the eye, which being confidered as quiefcent at S, the cotemporary pofitions of the fixed point P will be o, O, fuppofing So, SO refpectively parallel and equal to eP, EP. And becaufe $So = eP$, and $SO = EP$, and the contained angles are alfo equal,

equal, the line Oo will be equal to eE, and they will make equal angles with the diſtances.

23. The relative motion of a moving point, as ſeen from another which is alſo moved, is compounded of the real motion of the firſt point, and of a motion equal and contrary to that of the eye.

Fig. 3. Let the point move from B to L, while the eye moves from A to M; if the point were quieſcent at B, its apparent motion would be the ſame as if the eye were quieſcent, and the point had a real motion in a contrary direction along BE, equal and parallel to AM; therefore when the point alſo really moves along BL, the apparent motion will be the ſame, as if its motion were compounded of two real motions in the right lines BE, BL; it will therefore appear to move along the diagonal BN of the parallelogram $LBEN$.

24. The apparent motions of the points, as ſeen from each other, are equal and ſimilar.

25. If their real motions be ſimilar and oppoſite, their apparent motions are ſimilar to their real motions.

26. If equal motions, in parallel directions, be compounded with each of the real motions of a number of moving points, the relative motions of thoſe points are not changed.

Hence if the ſpace in which thoſe motions are performed, and to which they are always referred, be moved in any manner, the relative motions will not be changed.

LECTURE V.

1. PERCUSSION is the ſhock or colliſion of two bodies, which meeting alter each others motion.

The force of percuſſion is the ſame as the momentum or quantity of motion, and is meaſured by the product ariſing from the maſs moved, multiplied by the velocity; and that without any regard to the time, or duration of the action; for the action is conſidered totally independent of time, or as but for an inſtant, or an infinitely little time.

2. Percuſſion and Preſſure are heterogeneous forces, and do not admit of compariſon.

Let M denote any quantity of matter, having no motion or velocity, but ſimply preſſure; then will that preſſure be denoted by M itſelf; but its velocity being nothing, its percuſſive force will be $o \times M$, that is, nothing; and is therefore leſs than the ſmalleſt percuſſive force whatſoever. It is true, that we ſee motion produced by mere

 preſſure,

preſſure, to counteract which the oppoſition of a finite percuſſive force will be required; but then it muſt be obſerved, that the former has been an infinitely longer time than the latter in producing its effect. Hence it appears, that theſe two forces are related to each other, only as a ſurface is to a ſolid: by the motion of a ſurface through an infinite number of points, a ſolid is generated; and by the action of preſſure for an infinite number of moments, a quantity equal to a given percuſſive force is generated. See Hutton's Math. Dict. Art. Percuſſion.

It is to be remarked however, that in fact, motion can only be communicated by inſtantaneous impact, in the caſe of perfectly hard and inflexible bodies.

3. If a non-elaſtic body impinge directly on another non-elaſtic body that is at reſt, or moving the ſame way with leſs velocity, the ſum of the momentums will be the ſame before and after the impulſe.

4. When two non-elaſtic bodies impinge directly on each other with contrary motions, the ſum of the momentums after the impulſe is equal to their difference before it.

5. If two non-elaſtic bodies A and B move in the ſame right line, with the velocities x and y, and ſtrike each other, their common velocity v after impact will be $\frac{Ax \pm By}{A+B}$, where the ſign + muſt be

taken

taken when the bodies move in the ſame direction, and — when in oppoſite directions.

6. If A ſtrike B at reſt, the velocity after impact is $\frac{Ax}{A+B}$.

7. In the direct impact of two non-elaſtic bodies A and B, eſtimating the effects in the direction of A's motion, the velocity gained by B will be $=\frac{A}{A+B}\times$ relative velocity; and the velocity loſt by $A = \frac{B}{A+B}\times$ relative velocity, according as the bodies move in the ſame or oppoſite directions.

For the relative velocity, in the former caſe, is $x-y$; and v the common velocity is $=\frac{Ax+By}{A+B}$; therefore the velocity gained by $B = v - y = \frac{Ax+By}{A+B} - y = \frac{Ax-Ay}{A+B}$ $=\frac{A}{A+B}\times\overline{x-y}$: in the ſame manner is computed the velocity loſt by A.

Whilſt the relative velocity remains the ſame, the velocities gained by B and loſt by A remain the ſame.

The velocities gained by B and loſt by A are the ſame, whether both bodies be in motion, or A impinge on B at reſt, with a velocity equal to their relative velocity in the former caſe.

8. In the colliſions of elaſtic bodies, the velo-

cities loſt by A and gained by B will be twice as great, as if the bodies were non-elaſtic.

9. In the colliſions of elaſtic bodies, the difference of their velocities before and after impact remains the ſame.

Let $c =$ the velocity after impact, when the bodies are non-elaſtic and move the ſame way; then, when elaſtic, A's velocity after impact will be x — the velocity loſt by $A = x - 2\,\overline{x-c} = 2\,c - x$; and B's velocity after impact $= y +$ the velocity gained by $B = y + 2\,\overline{c-y} = 2\,c-y$; therefore the difference of velocities after impact $= x-y$.

In the ſame manner it is proved, when they move in contrary directions, that the difference of velocities after impact is $= x+y$.

10. The ſums of the products of the bodies into the ſquares of their reſpective velocities, before and after impact, are equal.

Let p and q be the velocities of A and B after impact, then, when the bodies move in the ſame direction, by the 3rd. law of motion, $Ax+By = Ap+Bq$; alſo $x-y = q-p$; hence $\left.\begin{array}{r} A\times\overline{x-p} = B\times\overline{q-y} \\ x+p = \quad q+y \end{array}\right\} \therefore A\times\overline{x^2-p^2} = B\times\overline{q^2-y^2}$; and $Ax^2+By^2=Ap^2+Bq^2$.

In the ſame mannner it is proved, when they move in contrary directions.

This equality between the ſums of the products of the bodies into the ſquares of their reſpective velocities before and after impact, was denominated by Bernouilli *Conſervatio Virium Vivarum*, and was conſidered as a general law: but

but it holds only in the case of elastic bodies, and is a consequence deducible from the 3rd. general law of motion, according to the Newtonian measure of motion.

11. In the collisions of two elastic bodies A and B, the velocities gained by B and lost by A will be $\frac{2A}{A+B}\times\overline{x\mp y}$, and $\frac{2B}{A+B}\times\overline{x\mp y}$ respectively, according as they move in the same or contrary directions.

12. When $A=B$, the bodies interchange velocities.

For in this case, $A+B=2A=2B$; therefore the velocities gained by B, and lost by A, are respectively equal to the relative velocity before impact.

If the bodies move in opposite directions, each will be reflected, having exchanged velocities.

13. If A strike B at rest, or $y=0$, B's velocity after impact will be $\frac{2Ax}{A+B}$, and A's will be $x-\frac{2Bx}{A+B}=\frac{Ax-Bx}{A+B}$.

If $A=B$, A will be at rest after the stroke; and B will move with the velocity which A had before impact.

If A strike an immoveable obstacle, that is, if B be infinite, and $y=0$, A's velocity after impact will be $=-x$, or A will be reflected back with the same velocity.

14. If there be any number of equal bodies, lying in the same right line, and the first strike the second, all the bodies will rest except the last, which

will

will fly off with the velocity of the first before impact.

15. If the bodies decrease in magnitude, each will go forward after the stroke.

For since A is greater than B, $\frac{Ax-Bx}{A+B}$ will be a positive quantity.

16. If they increase in magnitude, each will be reflected back, except the last, and the quantity of motion will continually increase.

For since A is less than B, $\frac{Ax-Bx}{A+B}$ will be negative; and $\frac{2\,Ax}{A+B}$ will be greater than x.

17. The increase will be a maximum, when the bodies increase in Geometrical progression.

Let A and C be two given bodies, between which is inserted the body X of intermediate magnitude; if the velocity of A be called a, the velocity of $C = \frac{2\,Aa}{A+X} \times \frac{2\,X}{X+C} = \frac{4\,Aa}{A+\frac{AC}{X}+X+C}$, a maximum; but the numerator is given, therefore this quantity will be greatest, when the denominator is least, that is, when $\frac{AC}{X}+X$ is least, because A and C are constant; now the rectangle $\frac{AC}{X} \times X$ is a given quantity, being equal to AC; therefore the sum of $\frac{AC}{X}$ and X is least, when they are equal, that is, when $\frac{AC}{X}=X$, and $AC=X^2$; that is, when X is a mean proportional between A and C.

18. If

18. If the number of mean proportionals interposed between two given bodies A and X, be increased without limit, the ratio of A's velocity to the velocity thus communicated to X will approximate to the ratio of $\sqrt{X}$ to $\sqrt{A}$ as its limit.

Let $A, B, C, D \ldots\ldots X$ be the bodies; $a, b, c, d \ldots x$. the velocities communicated to them. Then since the number of bodies is increased without limit, their differences will be diminished without limit: let $A + z = B$; then

$$2A + z : 2A :: a : b$$

$$\text{or } A + \tfrac{1}{2}z : A :: a : b$$

$$\text{but } A + \tfrac{1}{2}z : A :: \sqrt{A+z} : \sqrt{A} :: \sqrt{B} : \sqrt{A};$$

$$\text{therefore } \sqrt{B} : \sqrt{A} :: a : b;$$

in the same manner, $\sqrt{C} : \sqrt{B} :: b : c$ &c.

therefore, *componendo*, $\sqrt{X} : \sqrt{A} :: a : x$. See Parkinson's Mechanics, p. 181.

19. If the bodies be imperfectly elastic, and $m : n$ as the force with which they are compressed together, till they acquire a common velocity, to the force with which they separate, or as perfect elasticity to the imperfect elasticity of the given bodies; then if they move in the same direction, A's velocity after impact will be $x - \frac{m+n}{m} \times \frac{Bx - By}{A+B}$; and B's will be $y + \frac{m+n}{m} \times \frac{Ax - Ay}{A+B}$. If they move in contrary

contrary directions, the signs of the terms, where y enters, must be changed.

For if the bodies were inelastic, the velocity lost by impact by A would be $\frac{Bx-By}{A+B}$; and the velocity lost by elasticity will be $\frac{n}{m}\times\frac{Bx-By}{A+B}$; and their sum or $\frac{m+n}{m}\times\frac{Bx-By}{A+B}$ will be the whole velocity lost; therefore $x-\frac{m+n}{m}\times\frac{Bx-By}{A+B}$ will be the velocity of A after impact: and so of B.

Hence, if A strike B at rest, and be equal to it, B's velocity after impact $=\frac{m+n}{2m}\times x$; and A's velocity before : B's after : : $x : \frac{m+n}{2m}\times x$: : $2m : m+n$: : chord a described by A : chord b described by B; hence $m : n : : a : 2b-a$; and therefore by measuring these chords, the ratio of m to n, or the degree of elasticity of the bodies may be determined.

20. In the impact of bodies, the time in which velocity is communicated from one to another is finite.

At the first instant, the body struck will begin to move with a velocity less than that of the impinging body, the velocity of which will decrease, and that of the other will increase, as long as the impact causes a change in the figure of the two bodies; that is, till they shall have acquired

quired a common velocity, at which inſtant all accelerati- on ceaſes, if the bodies be not elaſtic. But if they be elaſ- tic, a new acceleration will then begin, ariſing from the force with which they recover their figure.

21. The motion of bodies is not cauſed by an immaterial, active fluid, which being incorporated with bodies, carries them along with it, in the direction in which it moves.

22. The Newtonian meaſure of motion furniſhes a general law in all caſes of colliſion, that the motions loſt and communicated are equal; the Leibnitzian meaſure does not.

It has been denied by ſome authors, that in the colliſions of bodies the ſame quantity of motion remains after the ſtroke, as before it; which is certainly true with reſpect to abſolute motion; for if equal non-elaſtic bodies meet with equal velocities, their abſolute motions after the ſtroke will be nothing; alſo if in a ſeries of elaſtic bodies increaſing in magnitude, motion be communicated from the firſt to the whole ſeries, they will all go back after the impact, except the laſt which will go forward, and the quantity of motion will continually increaſe; ſo that abſolute motion may be either entirely deſtroyed, or increaſed ad infinitum. But in relative motion, ſince matter poſſeſſes not in itſelf a power of creating or deſtroying motion, it follows, that as much as is gained by the body ſtruck, will be loſt by the other body; ſo that the quantity of motion, eſtimated in the ſame direction, will be the ſame before and after impact, eſtimating that as negative which is in a contrary direction.

 23. If

23. If a perfectly hard and smooth body impinge obliquely on another, the force must be resolved into two, one parallel and the other perpendicular to the plane which touches both bodies at the point of contact; the lateral force will have no effect whatever on the motion of the body struck, which will be impelled by the perpendicular force only.

If the surfaces penetrate a little, or slightly adhere at the point of contact, the perpendicular force will impell the body struck, and at the same time the lateral force will contribute to generate a rotatory motion round the centre of gravity.

24. When a perfectly hard body impinges obliquely on a perfectly hard and immoveable plane, after impact it will move along the plane; and the velocity before impact will be to its velocity after, as radius to the cosine of the angle which the direction of the impact makes with the plane.

25. The velocity before impact is to the velocity which is lost, as radius to the versed sine of the same angle.

For the difference between radius and the cosine of an angle is equal to the versed sine of the same angle.

26. If a perfectly elastic body impinge on a perfectly hard or elastic immoveable plane, it will be reflected in an angle equal to the angle of incidence.

The velocity after reflection is equal to the velocity before.

27. If

27. If two bodies, which are non-elaftic, impinge on each other obliquely, after impact they will defcribe the diagonal of a rectangular parallelogram, one of whofe fides, in the plane which touches both bodies in the point of impact, expreffes the fum of their motions in a direction parallel to that plane, and the other their difference in a direction perpendicular to it.

28. If the bodies be elaftic, they will be reflected; and each will defcribe the diagonal of a rectangular parallelogram, one of whofe fides, in the common contingent plane, expreffes the velocity which it had before impact, in a direction parallel to that plane, and the other the velocity in a direction perpendicular to it, which arifes from the direct collifion of the bodies, in that fame perpendicular direction.

LECTURE VI.

1. A REPULSIVE force exifts between the particles of elaftic fluids, and alfo of folids.

If diluted vitriolic acid be poured on pounded marble, a confiderable quantity of an elaftic fluid will be extricated, called fixed air; now the exiftence of an elaftic fluid proves, that the particles endeavour to recede from each other by a repulfive force. The repulfive force between the particles of folids appears from this, that bodies of no great weight lying upon one another, are not in actual contact, 1ft. Becaufe if an electric fhock be tranfmitted through them, the fpark will be vifible between them. 2dly. Becaufe, when pieces of metal lie freely on each other, the fufion of the parts through which the electric fhock paffes, when it enters the body or paffes out of it, is difcovered; but no fufion is perceived, when the pieces of metal are preffed together by a fufficient weight.

2. An

2. An attractive force subsists between the particles of non-elastic fluids.

1st. If two globules of quicksilver or water, lying on a smooth plane, be brought to touch, they immediately rush together, and form one drop. 2dly. Small portions of these fluids assume a spherical figure.

3. An attractive force subsists between the particles of solids.

On this principle is explained the cohesion of bodies, by which their parts are connected together. When you break a body, you only overcome this attraction; and could you join the parts exactly in the same manner, it would be as strong, as it was before. The soldering or glueing of bodies is only the bringing of the constituent particles so near that the attraction of cohesion may take place.

4. The attraction of cohesion is not caused either by rest, which was the opinion of Des Cartes; or by the pressure of the air, which was the opinion of Bernouilli.

The pressure of the air on a circle of an inch diameter is 11,78 pounds; and on a circle of half an inch diameter is less than three pounds; but the force requisite to separate two circles of lead of even less diameter, brought into close contact, is several times greater.

5. The attraction of cohesion is exerted between solids and fluids, with various intensity.

Thus water and quicksilver are attracted by glass; and the particles of water are more strongly attracted by glass than

than by each other, but the particles of quickſilver attract each other more ſtrongly than they are attracted by glaſs.

6. The attraction of coheſion is proportional, cæteris paribus, to the ſurfaces in contact.

If a drop of oil be placed between two glaſs planes forming a ſmall angle, their concourſe being parallel to the horizon, at different diſtances of the drop from the concourſe, the planes muſt be elevated to different heights to produce its quieſcence. Now ſince the attractive accelerating forces of the planes prevent the aſcent of the drop, they are equal to the accelerating forces of the drop on the plane; but theſe forces are as the heights of the plane, which by meaſurement are found to be inverſely as the ſquares of the diſtances of the drop from the concourſe. But the drop ſpreads over ſpaces inverſely as the diſtances from the concourſe; therefore the forces, by which the particles of oil, in contact with equal portions of the plane, are accelerated, are inverſely as the diſtances from the concourſe; now the abſolute attracting forces are as the accelerating forces and quantities of matter conjointly; but the quantities of matter are inverſely as the accelerating forces; therefore the abſolute forces of equal ſurfaces in contact are equal.

7. If two parallel glaſs planes be immerſed perpendicularly in water, it will riſe between them above the level of the external water.

This elevation of the water is cauſed by the attraction of the glaſs on a threefold account; 1ſt. The action of the glaſs upon the water immediately below the planes, which diminiſhes its gravity; whence it is preſſed upwards by the

the collateral columns of water, which are beyond the ſphere of activity of the glaſs. 2dly. The attraction of the loweſt films of glaſs, exerted on the water which lies between them. 3dly. The attraction of the films of glaſs which are exſtant above the ſurface of the water; the altitude of theſe acting films being always equal to the ſpace through which the attraction of glaſs on water extends.

8. If two glaſs planes forming an angle, be immerſed perpendicularly in water, the ſurface of the water which riſes between them will form a regular curve, the height of the water being always reciprocally as its diſtance from the concourſe of the planes.

This curve is the conical hyperbola, whoſe aſſymptotes are the concourſe of the planes, and a line on the horizontal level of the water, which biſects the angle of the planes.

9. If a capillary tube be immerſed perpendicularly in water, the water will riſe in the tube, and ſtand above the level of the ambient fluid in the veſſel.

This aſcent is accounted for in the ſame manner as between parallel glaſs planes.

10. The aſcent of water in capillary tubes is neither cauſed by the attraction of the whole interior ſurface of the tube, nor by the preſſure of the air.

11. The aſcent of water in capillary tubes is cauſed by the action of the loweſt glaſs annulus of the tube.

Becauſe

1st. Becaufe that is the only annulus whofe force in raifing the water is not counteracted by another annulus below it,drawing the water downwards. 2dly. If the water were fufpended by the annulus contiguous to the upper furface of the water, the water below the fphere of its action muft be fuftained either by the cohefive force of the water, or by the preffure of the air; but the cohefive force of the water is inadequate to this effect; and it does not depend on the preffure of the air, becaufe the experiment fucceeds equally well in vacuo.

12. If a tube be compofed of two capillary tubes of different diameters, the water will be fufpended to the fame height, whether the wide or narrow end be immerfed in the water; provided that when the narrow tube is immerfed, the water be above the juncture; but when the wider is immerfed, it be below it.

For the narrow tube being completely full, the equal annuli, at each extremity, attract the intermediate column of water with equal forces, in contrary directions, and therefore deftroy each others effects; the wide annnulus therefore, at the juncture, produces its effect undifturbed, and fuftains a column whofe magnitude is proportional to its diameter.

13. If when the wide tube is immerfed, the water be permitted to rife to the narrower, it will be fufpended at the fame height, as if the tube were continued of the fame bore with the narrower.

This

This phænomenon depends on the preſſure of the air, for it does not ſucceed in vacuo.

14. The height of the water in different capillary tubes is reciprocally as the diameter of the tube.

Let G denote the force with which glaſs attracts water, and F the force with which water attracts itſelf, then $G—F$ will denote the force, with which the water is elevated by the action of the lower annulus on the fluid juſt below the orifice of the tube. Likewiſe $G—F$ will denote the force with which the water within the tube, and in contact with the lower annulus, is elevated; and $2\,G—2\,F$ will denote the whole force, by which it is elevated on both theſe accounts together. But F is the force, by which it is raiſed by the action of the annulus, which is extant above the ſurface of the water; therefore $2\,G—F$ will expreſs the entire force, with which water is raiſed in a capillary tube of glaſs.

Hence the fluid will always riſe in a capillary tube, if the attraction which ſubſiſts between it and the fluid, be greater than half the force which ſubſiſts between the particles of the fluid themſelves.

If G and F be nearly equal, whatever may be their magnitude, the height of the fluid will be the ſame whether the tube continue immerſed in the fluid, or be lifted up out of it.

LECTURE VII.

1. If water be suspended in a capillary tube, and the tube be then inverted, the water will descend.

2. But in an upright conical tube it will continue suspended.

3. If the experiments hitherto recited, made with glass planes and capillary tubes, be instituted with mercury, the effects will be the reverse of those which are produced, when water is the fluid made use of.

Thus if a capillary tube of glass be immersed in mercury, the fluid within the tube will stand lower than the surface of the external mercury; and the depression will be reciprocally as the diameter of the tube; which depression is caused, 1st. By the action of the glass on the mercury immediately below the orifice of the tube, which attracting the mercury upwards less forcibly than it would be, if

if the tube were removed, it is comparatively heavier, and therefore ſubſides. 2dly. The cylinder of mercury at the very loweſt extremity of the tube, whoſe altitude is equal to that of the attracting annulus, is leſs attracted upwards, than if the glaſs were removed, and therefore the mercury ſubſides on this account alſo, as before. 3dly. The upper plate of mercury within the tube is actually neither attracted upwards nor downwards; but if the tube were removed, the upper plate of mercury would be attracted downwards by the force of an annulus of mercury; therefore the mercury, on this account, is lighter, than if the tube were removed, by the attraction of a mercurial annulus.

Let M denote the attractive force of an annulus of mercury on mercury; G the force with which an equal annulus of glaſs attracts mercury; then will $M—G$ denote the force, with which the mercury ſubſides by each of the two firſt cauſes; and $2M - 2G$ denotes the force with which it ſubſides by the action of both cauſes conjoined; but M, which denotes the force, with which it would aſcend by the operation of the third cauſe, being ſubtracted from $2M—2G$, the remainder $M - 2G$ will expreſs the entire force with which the mercury ſubſides, all cauſes being taken into conſideration.

Hence the fluid within the tube will ſtand lower than the level of the external fluid only when the attraction which ſubſiſts between its particles, exceeds the attraction which ſubſiſts between it and the tube, in a greater ratio than 2 to 1.

4. The phænomena of the Rope-Pump depend on corpuſcular attraction.

In the beſt contrived machine of this kind, a man by his utmoſt exertion can raiſe 9 gallons of water 95 feet high in a minute; but a man with a good common ſucking pump can raiſe 15 gallons to the ſame height in the ſame time; therefore the Rope-Pump, however ingenious, is of no great uſe in mechanics.

5. The phænomena of Blowing Machines depend likewiſe on corpuſcular attraction.

Theſe machines conſiſt merely of a vertical tube, through which deſcends a continued ſtream of water; the water carries down with it a good quantity of air, which being diſlodged from the water, and collected in a receiver, is conducted by tubes to any deſired part of the furnace.

LECTURE

LECTURE VIII.

1. THE mutual actions of masses of matter are regulated by the attraction of gravity, which acts with great force even at considerable distances.

All bodies in the neighbourhood of the earth have a tendency to move towards it, and in a direction nearly passing through its centre; this tendency is called Gravity, and arises from the compound tendency towards all its parts, for the direction of the combined action of all the particles composing a sphere, passes thro the centre.

The weight of a body is its tendency to the earth, compared with the like tendency of some other body, which is considered as a standard,

2. Gravity at all heights to which bodies are usually projected above the level of the sea, is found to be a constant force, or to act with the same intensity.

3. But accurately speaking, gravity at all distances above the surface of the earth, varies in the inverse

inverſe duplicate ratio of the diſtance from its centre.

4. Below the ſurface, gravity varies in the direct ſimple ratio of the diſtance from the centre.

5. All bodies in the ſame latitude, are found to be equally acted upon by gravity, whether quieſcent or moving, or whatever be their magnitude, figure, or denſity.

In the latitude of London all bodies acquire a velocity in falling 1″, which would carry them uniformly through 32,2 feet in the ſame time.

6. Suppoſing the earth ſpherical, the diminution of gravity, ariſing from the centrifugal force, varies as the ſquare of the coſine of latitude.

7. The figure of the earth is not exactly ſpherical, but is that of a ſpheroid flatted at the poles, and ſwelling out at the equator.

In the year 1736, the French Mathematicians were ſent towards the North Pole to meaſure a degree of the meridian; and they determined it to be 57405 French toiſes in the lat. 66° 20′; the Mathematicians who were ſent at the ſame time to Peru, for the ſame purpoſe, determined the length of a degree under the equator to be 56749 toiſes; whence it follows, that the curvature of the earth's ſurface is greater at the equator than at the poles; and conſequently, that the equatorial diameter is greater than the axis of the earth.

The

The menfurations which may be chiefly relied on, are thofe inftituted in Peru, at Turin, Vienna, in France, and in Lapland; not only becaufe of the fkill and care of the Mathematicians who conducted them; but likewife on account of the fituation of thofe places, which were fo circumftanced, that the plumb-lines of the inftruments were not difturbed by the attraction of neighbouring mountains.

The meafurements in thofe places were as follows:

1. Peru . . .	Lat.	0	—	56749 Toifes
2. Firft meafure in France	43°	31	—	57048
3. Turin. . . .	44	44	—	57138
4. Second meafure in France	45	45	—	57050
5. Vienna . . .	48	43	—	57086
6. Paris . . .	49	23	—	57074
7. Lapland . . .	66	20	—	57405

If thefe different meafures be varioufly combined, different ratios of the axes will arife, the mean of which appears to be the ratio of 230 to 231. It is admitted that the errors, in meafurement, may amount to 40 toifes in one degree; neverthelefs, as Frifi obferves, actual meafurement feems preferable to any calculation derived from the lengths of pendulums, becaufe the internal conftitution of the earth will have a confiderable influence on the times of their vibrations; it is alfo to be preferred to any calculation derived from the hypothefis of the earth's having been, at its formation, entirely fluid, becaufe obfervations on the inferior ftrata of the earth do not appear to juftify that hypothefis.

8. The

8. The degrees of the terrestrial meridian, in receding from the equator towards the poles, are increased in the duplicate ratio of the right sine of latitude.

Fig. 4. In the ellipse $MAmB$, let CA be the greater, and CM the less semi-axis; PN a perpendicular to the perimeter of the ellipse in P, meeting the axis; and PE an ordinate: then by Prop. 24. Lib. 2. and Cor. 1. Prop. 31. Lib. 1. Ham. Con. $PN^2 = (PE^2 + NE^2 =) \; CM^2 - \overline{CA^2 - CM^2} \times \frac{CM^2 . CE^2}{CA^4}$; and by Prop. 18. Lib. 5. and Prop. 22. Lib. 2 H. C. the radius of curvature at P is $= \frac{CA^2 \times PN^3}{CM^4}$. If instead of CE^2 we substitute $\frac{CA^2}{CM^2} \times \overline{CM^2 - PE^2}$, we shall have $PN^2 = \frac{CM^4}{CA^2} + \frac{\overline{CA^2 - CM^2}}{CA^2} \times PE^2$; and if the sine of the angle PNE be called t, we shall have $PN^2 = \frac{CM^4}{CA^2 - \overline{CA^2 - CM^2}} \times t^2$; and this being substituted, the radius of curvature at $P = \frac{CA^2 \times CM^2}{\overline{CA^2 - \overline{CA^2 - CM^2} \times t^2}|^{\frac{3}{2}}}$

If now we make $CM = c$, $CA = c + b$, and b, the difference of the semi-axes, be so small, that its square may be neglected, the radius of curvature will be $= \frac{c^4 + 2c^2b}{\overline{c^2 + 2cb - 2cb.s^2}|^{\frac{3}{2}}}$ $= c - b + 3b.t^2$. See Frisi's Cosmographia.

9. The

9. The oblate spheroidical figure of the earth may be determined from the measurement of a meridional degree in two different latitudes.

Fig. 4. For $PN^2 = \frac{CM^4}{CA^2 - \overline{CA^2 - CM^2} \times t^2}$, and the radius of curvature which is directly as PN^3, will be in the inverse sesquiplicate ratio of $CA^2 - \overline{CA^2 - CM^2} \times t^2$; if therefore the measurement of a degree be called G, and the sine of latitude t; and there be another arch E, nearer to the equator, and the sine of latitude be s, we shall have this analogy, $G : E :: \overline{CA^2 - \overline{CA^2 - CM^2} . s^2}|^{\frac{3}{2}} : \overline{CA^2 - \overline{CA^2 - CM^2} . t^2}|^{\frac{3}{2}}$; that is, since $CA = c + b$, and $CA - CM = b$, and b is very small, $G : E :: \overline{CA^2 - 2CA . b . s^2}|^{\frac{3}{2}} : \overline{CA^2 - 2CA . b . t^2}|^{\frac{3}{2}}$ very nearly; or $G : E :: CA - 3b . s^2 : CA - 3b . t^2$; whence $\frac{b}{CA} = \frac{b}{c+b} = \frac{G - E}{3G . t^2 - 2E . s^2}$. And if E be measured at the equator, $\frac{b}{c+b} = \frac{G - E}{3G . t^2}$. See Frisi's Cosmograph.

The figure of the earth might also be determined by the measurement of a degree in two parallels of latitude, but not so accurately as by meridional arcs, 1st. Because when the distance of the two stations, in the same parallel, is measured, the celestial arc is not that of a parallel circle, but is nearly the arc of a great circle; and always exceeds the arc truely correspondent to the terrestrial arc. 2dly. The interval of the meridians passing through the two stations must be determined by a time-keeper, a very small error in

the going of which will produce a very confiderable error in the computation.

10. The mutual gravitation of bodies in the neighbourhood of the earth is rendered only infenfible by the predominant influence of the earth.

For it has been afcertained, by the accurate experiments of Doctor Mafkelyne, that the attraction of a mountain is fufficient to draw the plumb line fenfibly from the perpendicular.

The quantity of attraction of a hill is meafured by the deviation of the plumb-line from the perpendicular; which deviation it found by making two obfervations of the zenith diftances of a fixed ftar, one on the north fide of the hill, the other on the fouth fide. On the fouth fide, the plumb-line being carried northwards at its lower extremity, towards the mountain, will occafion the apparent zenith diftance to approach the equator; and therefore the latitude of the place will appear too fmall by the quantity of attraction: the contrary happens on the north fide of the hill. Therefore the difference of latitudes will appear too great by the fum of the attractions; if therefore the difference of latitude of the two ftations determined by actual meafurement, according to the length of a degree of latitude in that parallel, be fubtracted from the difference of latitude determined by celeftial obfervations, the difference will give the fum of the two contrary attractions of the hill.

11. Gravity is not caufed by the emiffion of magnetic effluvia; nor by the impulfe, trufion, or action of the materia fubtilis.

12. Gravity

12. Gravity is not an innate property or primary quality of body.

13. Gravity cannot be accounted for by the action of any fluid, whether elaſtic or non-elaſtic, quieſcent or agitated.

However unſearchable the efficient cauſe of gravity may be, its final cauſe is evident, 1ſt. To prevent the diſſipation of the parts of the planets, which would be cauſed by their rotation on their axes. 2dly. To produce a viciſitude of ſeaſons, a revolution of the planets round the ſun was neceſſary; which is effected by the combination of gravity with the projectile force.

LECTURE IX.

1. TERRESTRIAL mechanics are generally refolved into two claffes, Statics, which treat of the equilibrium of bodies, and Dynamics, which treat of their motion.

2. A lever is defined to be an inflexible right line void of gravity, and moveable about a fixed point called the fulcrum; but as every body is ponderous, the lever is balanced before the weights are applied, in order that the weight of the inftrument itfelf may not difturb the equilibrium, which fhould be produced according to theory.

3. If a weight be fupported on an horizontal lever which refts on two fulcrums, and be fimilarly fituated with refpect to them both, each of the fulcrums will fuftain a preffure equal to half the weight.

For fince the lever is fuppofed to be horizontal, the whole weight preffes perpendicularly againft it, and therefore the two

two fulcrums ſuſtain a perpendicular preſſure equal to the whole weight; but they are ſimilarly ſituated with reſpect to the weight, therefore the preſſure is equally diſtributed between them.

4. Equal forces acting perpendicularly at the extremities of equal arms of a lever will balance each other.

5. If a uniform cylinder be ſupported on the middle point of its axis, in an horizontal poſition, it will continue at reſt.

6. If a uniform cylinder be balanced on the middle of an horizontal lever, the equilibrium will not be diſturbed, if the cylinder be cut into any two parts, by a plane perpendicular to the axis of the cylinder.

For the force of coheſion can act only into two ways, either in preventing the particles, which are in contact, from ſeparating by moving in a vertical direction, or in drawing them horizontally towards each other; now the former effect is equally produced by the lever, on which the cylinder reſts; and ſince the latter conſiſts merely in drawing the particles in a direction parallel to the lever, it can have no effect in moving it.

7. If a uniform cylinder reſt on an horizontal lever, which is ſupported on a fulcrum at one of its extremities, it will be balanced by half the weight of the cylinder, at twice the diſtance of the middle point of the cylinder from the fulcrum.

Fig.

Fig. 5. For let C be the fulcrum, and AD the cylinder, whose middle point is E; take $EQ = EC$, and suppose the end Q to be sustained by a prop; then as AD is similarly situated with respect to each end of the lever, the prop and fulcrum must bear equal parts of the whole; and therefore the prop at Q will be pressed with a weight equal to half that of the cylinder; therefore the cylinder will be balanced by half its weight pulling the point Q upwards, that is, by the same half weight hung from the point P, pulling it downwards, if $PC = CQ = 2CE$.

8. Any two weights acting perpendicularly upon an horizontal lever, and at contrary sides of the fulcrum, will balance each other, if they be reciprocally proportional to their distances from the fulcrum.

Fig. 6. Let x and y be the two weights, and ef the lever; let each of the two weights be doubled, and formed into a uniform cylinder AB, whose length is equal to ef; bisect AB in C, then this cylinder will balance itself on the fulcrum C; let it be cut at D, so that $AD : DB :: x : y$; then bisect AD in E, and DB in F, so will $AE = x$, and $DF = y$; and the equilibrium will still remain; let $Ee = EC$, and $Ff = FC$; then $eA = DC = fB$. Now a weight equal to AE or x, at the distance eC from the fulcrum, would balance the cylinder AD; but AD balances the cylinder DB; therefore x at the distance eC would balance the cylinder DB. In like manner, half the cylinder DB or y, at the distance Cf from the fulcrum, would balance DB; therefore since the weights x and y, at their respective distances

tances eC, fC from the fulcrum would balance the same weight, in the same circumstances, they will balance each other. But $eC = DB$, and $Cf = AD$, therefore $eC : Cf :: DB : AD :: y : x$; that is, the weights x and y, when in equilibrium, are to each other reciprocally as their distances from the fulcrum.

9. There will be an equilibrium on the lever, if the weights be reciprocally as the velocities, with which they would move, if they were set in motion.

10. It is of no consequence in what point of the line of direction the force acts.

11. When an inflexible line is kept in equilibrium by three forces acting on it, any two of these forces will be to each other reciprocally as the perpendicular distances of the lines of direction from the point of application of the third force.

12. There will be an equilibrium on the compound lever, when the power is to the weight, in a ratio compounded of the ratios of the several powers to their respective counterpoising weights on each lever separately.

LEC.

LECTURE X.

1. ANY of the five regular ſolids being ſupported on its centre of magnitude, will continue at reſt in any poſition.

2. There is a point within every body which being ſupported, the body will continue at reſt in any poſition.

This point is called the centre of gravity. Though all bodies have a centre of gravity, or a point through which a plane paſſing in any direction reſolves the body into two equiponderating parts; yet all bodies have not a centre of magnitude, or a point through which a plane paſſing in any direction will reſolve the body into two equal magnitudes. Thus no ordinate polygon, of an uneven number of ſides, has a centre of magnitude.

3. If

3. If two bodies be connected by a right line joining their centres of gravity, and the point which divides that line in the reciprocal proportion of their weights, be made a fulcrum, the bodies will continue in equilibrium in any position.

This point is called their common centre of gravity. Hence, the effect of a body to move a lever is the same, as if its weight were concentrated in its centre of gravity. And if a weight be suspended from any point of a lever, its effect to move it is the same, as if the weight were concentrated in that point.

4. The common centre of gravity of three bodies is the common centre of gravity of any one, and of the other two considered as one body united in their common centre of gravity. And so on for any other number of bodies.

5. If from two angles of a triangle there be drawn right lines bisecting the opposite sides, the centre of gravity will be in this line, at two thirds from the vertex.

6. All plane figures may be resolved into triangles, whose centres of gravity being found by the last article, their common centre of gravity will be the centre of gravity of the figure.

7. A body or system of bodies cannot have more than one centre of gravity.

For if there were two, let us ſuppoſe the body to revolve on one of them, until the other lie in the ſame horizontal line; then it would follow, that the equilibrium in the lever would not be diſturbed by increaſing one of the arms and diminiſhing the other; which is abſurd.

8. A force applied at the centre of gravity of a body cannot produce a rotatory motion.

For every particle reſiſts, by its inertia, the communication of motion, and in a direction oppoſite to that in which the force applied tends to communicate the motion; their reſiſtances therefore act in parallel lines, in the ſame manner as their gravities; and they are proportional to their weights; therefore ſince the weights balance each other on the centre of gravity, ſo alſo will the reſiſtances of the inertia of the particles.

9. When two weights are ſuſpended by a cord, or otherwiſe hang freely from a ſtraight lever, if there be an equilibrium in the horizontal poſition, there will alſo be an equilibrium however it may be inclined to the horizon.

10. If the weight be faſtened to the lever, though there may be an equilibrium when the lever is horizontal, yet there will not, if it be inclined.

Hence, when two men carry a weight by a lever, from which it hangs freely, there will be no advantage gained, nor inconvenience ſuffered either by the firſt man or the laſt in going up or down hill. But if the weight be faſtened to the lever, and reſt upon it, in going up hill the foremoſt

man

man will fuftain a lefs portion of the weight, and in going down hill will fuftain a greater. And the contrary will happen, when the weight is attached to the lever, and below it.

11. The centre of gravity continually endeavours to defcend in a vertical line.

Gravity acts in a direction perpendicular to the horizon; and fince the whole weight of the body may be confidered as collected into the centre of gravity, that point will always endeavour to defcend in a vertical line, and with a force equal to the body's weight; for this reafon, a vertical line, paffing through the body's centre of gravity is called the line of direction.

12. If two rulers forming an angle at one of their ends, reft there on an horizontal plane, whilft at the other they are raifed above the plane, and if a body confifting of two equal and fimilar cones, joined at their bafes, be laid upon the rulers in fuch a manner, as that the edge of their bafes may lie between the rulers, it will, when left to itfelf, roll towards the elevated extremities of the rulers, provided the femidiameter of the cone be greater than the height of the rulers, where their interval is equal to the axis of the double cone.

If the femidiameter of the double cone be equal to that altitude, it will continue at reft on any part of the rulers; if lefs, it will roll down.

The contrary will happen if the cones be united at their vertexes.

13. When the centre of gravity and the centre of magnitude of a cylinder do not coincide, it can roll up an inclined plane, if the right ſine of the angle of elevation of the plane be leſs than the interval between the centres of gravity and magnitude, the ſemidiameter of the cylinder being radius; if equal, it will continue at reſt in one poſition; if greater, it will roll down the plane in every poſition.

14. If the centre of gravity of a body be beneath any point of it, which is ſupported, the body will remain ſupported, otherwiſe not.

Hence, if a body be increaſed or diminiſhed by the addition or ſubduction of any part, the poſition of the centre of gravity will be changed, and the body, which before was ſupported, may now fall; and the contrary.

15. If the plane baſe of a body be placed on an horizontal ſurface, the line of direction paſſing without the baſe, the body will fall; but if the line of direction paſs through the baſe, the body will remain ſupported.

16. If a body be ſet upon an inclined plane, the line of direction will fall oblique to the baſe; in this caſe, if the line of direction fall within the baſe, the body will ſlide down the plane; but

but if it fall without the base, the body will roll, or partly slide and partly roll, according as the quantity of friction is greater or less.

If there were no friction, the body of whatever form would slide without rolling.

LEC-

LECTURE XI.

1. IF a body hang freely from a centre of ſuſpenſion, it will not continue at reſt, unleſs the line of direction paſs through the centre of ſuſpenſion.

When the line of direction does not paſs through the centre of ſuſpenſion, the weight of the body may be always reſolved into two forces, one of which impells the centre of gravity directly from the centre of ſuſpenſion, and is totally counteracted, becauſe that centre is fixed; the other force acting in a direction perpendicular to the former, and having no counterpoiſe, urges the centre of gravity in the direction of a tangent to the circle, which that point deſcribes round the centre of ſuſpenſion. This force is always as the ſine of the angular diſtance of the centre of gravity from the loweſt point.

If

If a body be fuſpended by a line from a fixed point, it will be at reft, only when the line is perpendicular to the horizon. An inſtrument of this kind is called a plumb-line; by means of this line the vertical direction, and the horizontal plane which is perpendicular to it, are determined; for two lines which interfect each other being determined, the plane paſſing through them is alſo determined; if therefore there be two lines, which are perpendicular to each other, you can by the plumb-line aſcertain, when one of them is vertical, and conſequently the other horizontal; and this being done in a ſecond poſition of the lines, ſo as to give a ſecond horizontal line cutting the former, the horizontal plane paſſing through them is determined. A plumb-line applied to right lines ſo ſituated, conſtitutes a level. An horizontal plane is alſo determined by ſpirit levels, which indeed have the advantage of being free from any agitation produced by the air; nevertheleſs the plumb-line is to be preferred, as not requiring any adjuſtments, and being eaſily conſtructed.

2. If a body be ſuſpended from centres of ſuſpenſion paſſing through different points of the body, the interſection of lines drawn from theſe centres perpendicular to the horizon, when the body is quieſcent, will coincide with the centre of gravity.

3. The centre of gravity of a body is ſometimes

not

not within the body itſelf, yet the ſame regard is to be had to its motion or ſupport, as if it were.

The centre of gravity of the bones of animals does not lie in the bones themſelves, but in their imaginary axes; in which conſtruction is manifeſted the wiſdom of the Divine Artiſt. The number and adheſion of the fibres compoſing a bone being given, the ſtrength of a ſolid cylinder of bone is to the ſtrength of a hollow cylindrical bone, directly as their diameters.

4. The diſtances of bodies, as far as relates to the action of their weights, is expreſſed by the diſtances of their centres of gravity; and the eſtimation of the forces, by which bodies or ſyſtems of bodies are urged by gravity near the ſurface of the earth, is determined by this point.

For the effect of the preſſure of the whole body to generate motion in a lever, is the ſame, as if all the matter were collected in the centre of gravity.

5. If any number of bodies be placed on a lever, there will be an equilibrium, when the products of the ſums of the bodies on each ſide of the fulcrum, multiplied into the diſtance of their common centre of gravity from the ſame fulcrum, are equal,

6. There will be an equilibrium in the lever, when the ſums of the products on each ſide, ariſing from the multiplication of each body into its reſpective diſtance from the fulcrum, are equal.

7. If

7. If a body equal to the ſum of any two bodies be placed in their centre of gravity, and the ſame quantities of motion in the ſame directions be communicated to it, which are communicated to the two bodies, this body will move in the ſame line, which the centre of gravity of the two bodies deſcribes, and with the ſame velocity.

Fig. 7. Suppoſe the momentum communicated to A, would cauſe it to move from A to x, in a given time; at x let the body be ſtopped; join AB, and xB; let E be the common centre of gravity of A and B, draw EF parallel to Ax; and, by the motion of A, the centre will have deſcribed the line EF. And if E be a body equal to $A+B$, and the ſame momentum be communicated to it, that was before communicated to A, and in the ſame direction, that is, in the direction EF, ſince the quantities of motion communicated to A and E are equal, their velocities will be reciprocally proportional to their quantities of matter, or E's velocity : A's velocity :: $A : A+B :: EF : Ax$; therefore EF and Ax are the ſpaces deſcribed by E and A in equal times, or EF will be the ſpace deſcribed by E in the given time. In the ſame manner, if the momentum communicated to B, would cauſe it to move from B to y in the ſame given time as before, then will the body E, in the centre of gravity, in that time, deſcribe the line EG, parallel to By. If now the momenta, inſtead of being communicated ſeparately, be communicated at the ſame inſtant, to the bodies, at the end of the given time they will be found at x and y; and E equal to the ſum of the

bodies in the common centre of gravity, will have defcribed the diagonal of the parallelogram, whofe conterminous fides are *EF*, *EG*.

Hence equal and contrary motions communicated to any fyftem of bodies will have no effect upon their centre of gravity. For the fame reafon, the motion of the centre of gravity of any number of bodies is not difturbed by their collifions.

Hence alfo, the centre of gravity of a fyftem of bodies will not be difturbed by their mutual attractions, as the motions thus communicated are always equal and oppofite. Hence alfo the centre of gravity of the folar fyftem is either at reft, or moves on uniformly in a right line.

8. If two bodies move uniformly in two right lines, their common centre of gravity will either be at reft, or move uniformly in a right line.

This follows directly from the laft article.

9. If one body be at reft, and another defcribe any curve, the centre of gravity will defcribe a fimilar curve.

10. If one body be moved in any direction, the others continuing at reft, the correfponding motion of the common centre of gravity, eftimated in any given direction, will be to the motion of the aforefaid body, eftimated in the fame direction, as the weight of the body moved is to the weight of the whole fyftem.

11. The particles of a fyftem, which are acted on only

only by their mutual attractions, will meet in their common centre of gravity.

For they muſt meet, and the common centre of gravity remains at reſt.

12. The perpendicular diſtance of the centre of gravity of any number of bodies from a given plane, is not changed by the motion of the bodies in planes parallel to that given plane.

13. The ſum of the products of any number of bodies multiplied into their perpendicular diſtances from any propoſed plane is equal to the product of the ſum of the bodies multiplied into the perpendicular diſtance of their common centre of gravity from that plane.

Hence, if the ſum of the products of all the bodies into their reſpective diſtances from any propoſed plane be divided by the ſum of all the bodies, the quotient will be the diſtance of the centre of gravity from the propoſed plane; and thus may the poſition of the centre of gravity be determined by calculation.

LECTURE XII.

1. IF a beam be so constructed, that the line joining the centre of gravity and the fulcrum shall be perpendicular to any given line in the beam, the beam hanging freely on its fulcrum, will rest only when that given line is horizontal.

Upon this principle is constructed the balance, which is a lever with equal arms, and whose fulcrum or centre of motion is situated immediately above the centre of gravity of the beam, when horizontal. Its use is to compare the weights of bodies with each other.

2. The axis of motion of the balance should be above the centre of gravity of the beam.

The horizontal position of the beam when loaded, in the comparing of weights, is determined by an index passing through the centres of gravity and motion, and perpendicular to the axis of the beam.

3. When

3. When the balance unloaded is quiefcent, and therefore horizontal, if the index which paffes through the fulcrum be directed to any fixed point, and again, when the balance is reverfed, it be directed to the fame fixed point, it is in the right line which joins the centre of gravity and the fulcrum.

By this means the pofition of the index is adjufted,

4. The perpendicular diftances of the points of application of the weights to be compared, from the right line which joins the centres of gravity and motion, fhould be equal, that is, the arms of the balance ought to be equal.

5. The points of application from which the weights are fufpended, fhould be in the fame right line perpendicular to the line joining the centres of gravity and motion.

6. The nearer the right line joining the points of application is to the centre of motion, a given difference of weight will produce larger vibrations of the balance, and a more fenfible effect.

7. If the centre of motion be fituated below the line joining the points of application, the beam, when loaded with equal weights, will overfet, reft in any pofition, or equilibrate according to the weight.

8. When

8. When two given weights, suspended from the arms of a balance, are in equilibrium, if these weights, when transferred to the opposite scales, be still in equilibrium, the arms of the balance are equal.

9. The various adjustments of the balance are these: 1st. Equal weights are readily found, whatever be the state of the balance; for if they reduce the beam to the same position, when successively applied to the same arm, they must be equal: then if these equal weights transposed do not disturb the position of the beam, the arms are equal. 2dly. If unequal weights transposed produce equal deflections of the beam, the points of suspension are in the same right line, perpendicular to that which joins the centres of gravity and motion; and therefore the line joining these points will be horizontal, when the beam hangs freely. 3dly. Let the index be directed to any fixed point, then the beam being reversed, if it still pass through the same point, the the index is perpendicular to the axis of the beam.

Weights are made by the subdivision of a standard weight: these standard weights are the pound Troy, and the pound Avoirdupois, which are different, and differently subdivided.

TROY

TROY WEIGHT.

As used by Apothecaries.

Grains.	Scruples.	Drams.	Ounces.	Pounds.
20 =	1℈			
60 =	3 =	1ʒ		
480 =	24 =	8 =	1℥	
5760 =	288 =	96 =	12 =	1lb.

As used by Goldsmiths.

Grains.	Pennyweight.	Ounces.	Pounds.
24 =	1 dwt.		
480 =	20 =	1oz.	
5760 =	240 =	12 =	1lb.

AVOIRDUPOIS WEIGHT.

Drams.	Ounce.	Pound.
16 =	1	
256 =	16 =	1

The Avoirdupois pound contains 7004 grains Troy; whence it exceeds the Troy pound nearly in the ratio of 107 to 88; but as it is divided into 16 ounces, whereas the Troy pound is divided only into 12, the Troy ounce exceeds the Avoirdupois ounce nearly in the ratio of 80 to 73, the former containing 480 grains, and the latter 437, 75. The Avoirdupois weight is used in weighing gross substances; Troy weight is used for philosophical purposes. If the standard weight be continually halved, it will produce the common pile, which is the smallest number for weighing between its extremes, without placing any weight in the scale with the body under consideration. Mathematicians have computed the least possible number of weights with which the weight of any substance, within given limits, may be estimated; but this determination is of little use, because with so small a number, it must often happen, that the scales will be loaded with weights on each side, whose

whose difference only is to be ascertained. We are not therefore to look for the least possible number of weights, by which the weighing of bodies may be effected, but that number and arrangement by which it can be done with the greatest facility. Now as philosophical computations are chiefly made in decimal fractions, it is most convenient to arrange weights in that progression; of which there should be two sets, one of ounces, the other of grains, in the following orders. See Atwood's Analysis.

Weights.		Ounces.	Weights.		Grains.
4	each of	100	1	of	1000
1	of	50	1	of	500
4	of	10	4	of	100
1	of	5	1	of	50
4	of	1	4	of	10
1	of	.5	1	of	5
4	of	.1	4	of	1
1	of	.05	1	of	.5
4	of	.01	4	of	.1
1	of	.005	1	of	.05
4	of	.001	4	of	.01

10. When the equiponderating arms of a balance are of unequal lengths, if two suspended weights be in equilibrium, when these weights are transposed, the equilibrium will be destroyed.

This is called a false balance, as not giving the true weight. But the true weight will be a geometrical mean between the two false weights. If the arms be very nearly equal, the true weight will be expeditiously found, with sufficient

ficient accuracy, by taking an arithmetical mean between the two falſe weights.

11. If different weights ſucceſſively ſuſpended at the ſame diſtance from the fulcrum of a ſtraight lever, be counterpoiſed by the ſame weight at different diſtances from it, theſe weights will be to each other, as the diſtances of the invariable weight from the fulcrum.

On this principle is conſtructed the Statera Romana, or Steel-yard. This inſtrument is very convenient, becauſe it requires but one weight, is eaſily carried, and the preſſure on the fulcrum is leſs than in ſcales, when the ſubſtance to be weighed is heavier than the conſtant weight. But it is not ſo exact as ſcales in determining the equality of ſmall weights; 1ſt. Becauſe, the length of the beam being given, the arms in the Statera Romana will be ſhorter than in the balance, whenever the weight is equal to or leſs than the conſtant weight. 2dly. The balance is capable of nicer adjuſtment. 3dly. The ſubdiviſion of weights can be effected with greater preciſion than the ſubdiviſion of the arm of the Statera Romana. 4thly. The preſſure on the fulcrum is greater in the ſteel-yard than in ſcales, when the conſtant weight is greater than that of the ſubſtance to be weighed.

12. If a man ſtanding in one ſcale and counterpoiſed by a weight in the other, lay his hand to any part of the beam, and preſs it upwards, he will thereby deſtroy the equilibrium, and cauſe the ſcale in which he ſtands to preponderate: and the contrary will happen, if he pull it downwards.

LECTURE XIII.

1. IN every machine whatſoever, the power and weight will be in equilibrium, when they are to each other reciprocally as the velocities, with which they would move, if they were ſet in motion.

2. In the fixed pulley there is an equilibrium, when the power is equal to the weight.

3. In a ſingle moveable pulley there will be an equilibrium, when the power is to the weight as one to two.

4. If the ſame ſtring go round two ſets of pullies in two blocks, the power is to the weight, in caſe of equilibrium, as unity to the number of ſtrings at the lower block.

5. In a ſyſtem of one fixed and two moveable pullies, one of which aſcends as the other de-ſcends,

ſcends, where a ſtring, attached to the aſcending pulley, paſſes round the fixed pulley and is attached to the deſcending one, and another ſtring attached to the fixed pulley paſſes round both the moveable pullies, the power is to the weight, in caſe of equilibrium, as one to four.

6. In a ſyſtem of two fixed and two moveable pullies, one of which aſcends as the other deſcends, where a ſtring attached to the aſcending pulley paſſes round one of the fixed pullies and is attached to the deſcending one, and another ſtring attached alſo to the aſcending pulley, paſſes round the other fixed and both the moveable pullies, the power is to the weight, in caſe of equilibrium, as one to five.

The two preceding ſyſtems are called Spaniſh Burtons.

7. In a ſyſtem of moveable pullies, in which a ſeparate ſtring goes round each pulley, there will be an equilibrium, when the power is to the weight, as unity to that power of two of which the index is the number of moveable pullies.

8. In a ſyſtem of moveable pullies, in which the ſtring that goes round each pulley is fixed to the weight, there will be an equilibrium, when the power is to the weight, as unity to that power of two, of which the index is the number of pullies, the whole being diminiſhed by unity.

LECTURE XIV.

1. THERE will be an equilibrium in the wheel and axle, when the power is to the weight, as the radius of the axle to the radius of the wheel.

When the rope which goes round the axle is long, regard muſt be had to its weight, which is often very conſiderable, as in deep ſhafts in mines: in ſuch caſes it is generally counterpoiſed by a heavy chain, which is raiſed up or let down, according as the rope lengthens or ſhortens, by means of a cord wound round an axle, whoſe diameter is to that of the axle to which the rope is attached, as the length of the chain to the length of the rope.

2. If the diameter of the wheel increaſe in the ſame proportion in which the power diminiſhes, the force with which the wheel will continue to be turned, will always be of the ſame magnitude.

This

This principle is ingeniously applied in the action of the main spring on the fuzee of watches; and of the main spring on the tumbler of gun-locks.

3. In a machine consisting of several wheels so constructed, that the periphery of one may act upon the axle of another, there will be an equilibrium, when the power is to the weight in a ratio compounded of the ratios of the diameters and synchronal revolutions of the last axle and first wheel.

The number of synchronal revolutions performed by a wheel and the arbor that works it, is found by dividing the number of teeth in the wheel by the number of teeth in the pinion. The numbers of the teeth which play together should be prime to each other, that the same teeth may meet as seldom as possible.

4. When a toothed wheel works another, the peripheries of both wheels move with the same velocity.

5. If a toothed wheel whose diameter continues always parallel to itself, be moved round another, which turns on a fixed centre, the number of synchronal revolutions of the revolving and central wheel will be to each other, as the semi-diameter of the central wheel to the sum of the semi-diameters of the central and revolving wheels.

Fig. 8. Let the toothed wheel *AEB* whose diameter *AB* keeps always parallel to itself, be moved round the toothed wheel *DFG*, which turns on the fixed centre *C*; it is evident, that

that the centre K of the travelling wheel deſcribes a circle round the centre C; and conſequently, ſince the diameter AB is always parallel to itſelf, every point in the plane of the circle AEB will deſcribe equal and ſimilar figures, that is, every tooth of the travelling wheel will deſcribe a circle, whoſe radius is equal to KC. But a tooth of the central wheel DFG is always engaged with a tooth of the wheel AEB, and conſequently they move equally faſt; therefore the number of revolutions performed in a given time, will be inverſely as the ſemi-diameters of the circles; that is, while AEB performs one revolution, DFG will perform a number of revolutions which is to unity, as KC to EC.

When the radius of the travelling wheel vaniſhes, that wheel becomes a crank, and the number of ſynchronal revolutions of the crank and wheel equal.

LECTURE

LECTURE XV.

1. WHEN a refifting body is fuftained againft the face of a wedge, by a force acting at right angles to its direction, the power is to the refiftance, in cafe of equilibrium, as the fine of the femiangle of the wedge to the fine of the angle which the direction of the refiftance makes with the face of the wedge; and the fuftaining force will be as the cofine of the latter angle.

Fig. 9. Let *ABC* be a rectangular wedge, whofe angular point is *C*, face *BC*, and back *AB*. Let this wedge flide freely along the plane *LN*; let a body *E* be drawn or urged in the direction *KE* againft the face of the wedge, and let it be kept in that direction by a force acting in the direction *DE*, at right angles to *KE*. There are now three forces acting on the body *E*, viz. the refifting force *KE*, the fuftaining force *DE*, and the reaction of the wedge in the direction *AE*, perpendicular

lar to the surface *BC*. On *ED* let fall the perpendicular *AG*; and since the three forces are in equilibrium, they will be to each other as the sides of the triangle *AEG*, drawn parallel to their directions. Draw *EF* perpendicular to *AC*, and the force *AE* will be resolved into two; one *EF* pressing the wedge perpendicularly against the plane *LN*, which is balanced by the reaction of the plane; the other *FA*, which endeavours to move the wedge upwards along the plane *LN*, and which is balanced by the power on the back of the wedge. If therefore *AG* represent the force *KE*, *EG* will be the sustaining force, and *AF* the power applied on the back of the wedge, when these forces balance each other. And making *AE* radius, *AF* is the sine of the angle *AEF* or *ACB*; and *AG* is the sine of the angle *AEG* or *KEC*, these two angles being the complements of *AEK* to a right angle.

If the wedge be in form of an Isosceles triangle, composed of two rectangular wedges, the force *EF* which in the former case was counteracted by the plane, will now be counteracted by the other half of the wedge; and the power, resistance, and sustaining force will remain in the same ratio as before.

When *EK* is parallel to *BA*, *AG* becomes equal and parallel to *EF*; and *EG* equal and parallel to *AF*; and the power is to the resistance as *AF* to *EF*, or *AB* to *AC*, and equal to the sustaining force.

If *EK* be perpendicular to *AB* the back of the wedge, the direction of the resisting force will be parallel to *AB*; therefore this, in fact, is the same case with the former, the resisting and sustaining forces changing denominations.

When

When KE is perpendicular to BC, the ſine of the angle KEC is radius; and its coſine, which repreſents the ſuſtaining force, vaniſhes; therefore the power is to the reſiſtance, as the ſine of the ſemiangle of the wedge to radius. See Ludlam on the Wedge.

2. When the reſiſtance is made againſt the face of a wedge by a body which is not ſuſtained, but will adhere to the place to which it is applied without ſliding, the power is to the reſiſtance, in caſe of equilibrium, as the coſine of the difference between the ſemiangle of the wedge and the angle, which the direction of the reſiſtance makes with the face of the wedge, to radius.

Fig. 10. From any point K draw the line KE through the middle point of the back, meeting the face of the wedge in E; let E be the unſliding body, which acts in the direction EK; and let the magnitude of the force with which it is urged, be repreſented by AE; from E let fall the perpendicular EF upon AC; and the force AE will be reſolved into two, one of which EF will be balanced by the oppoſite half of the wedge, and the other AF will be counteracted by the power; therefore the power is to the reſiſtance as AF to AE, that is, making AE radius, as the coſine of the angle EAF to radius.

When KE is perpendicular to BC, the power is to the reſiſtance as AF to AE, that is, as the ſine of the ſemiangle of the wedge to radius.

When KE is parallel to AB, AF vaniſhes, that is, the power is indefinitely leſs than the weight.

When KE is perpendicular to AB, EF vaniſhes, and AF and AE, that is, the power and reſiſtance become equal.

3. When the reſiſting body is neither ſuſtained, nor adheres to the point to which it is applied, but ſlides freely along the face of the wedge, the power is to the reſiſtance, as the product of the ſines of the ſemiangle of the wedge and the angle in which the reſiſtance is inclined to its face, to the ſquare of radius.

Fig. 11. Let AE be perpendicular to BC, and let the body E be urged againſt the face of the wedge in the direction KE; and let KE repreſent the magnitude as well as the direction of that force. On AE produced let fall the perpendicular KO, which will be parallel to BC; thus will the force KE be reſolved into two, one of which KO will carry the body down along the face of the wedge, and the other OE will urge it perpendicularly againſt it. Now the power, in caſe of equilibrium, is to OE, that part of the reſiſtance which acts perpendicularly againſt the face of the wedge, as the ſine of the angle ACB to radius; and OE is to the whole reſiſtance, as OE to KE; that is, making KE radius, as the ſine of the angle OKE, or its alternate KEB, to radius. Therefore *ex æquo & componendo*, the power is to the reſiſtance, as ſine ACB $\times$ ſine KEB to the ſquare of radius.

When KE is perpendicular to BC, the ſine of the angle in which the reſiſtance is applied, is radius; therefore the power

power is to the reſiſtance as ſine $ACB \times$ radius to ſq. radius, that is, as the ſine of the ſemiangle of the wedge to radius.

When KE is parallel to AB, the angle of inclination is the complement of the ſemiangle of the wedge; and therefore the power is to the reſiſtance, as the product of the ſine and coſine of the ſemiangle of the wedge to the ſquare of radius.

When KE is perpendicular to AB, the angle of inclination is equal to the ſemiangle of the wedge, and the power is to the reſiſtance in a duplicate ratio of the ſine of the ſemiangle of the wedge to radius.

The theory of the equilibrium of the wedge is not of very great uſe in practical mechanics, becauſe the wedge is ſcarcely ever otherwiſe urged than by percuſſion. In cleaving of wood, the reſiſtance oppoſing the force of the mallet (ſuppoſing the ſides of the wedge perfectly poliſhed, and its edge a mathematical line) is the coheſion of the particles of the wood to be ſeparated; and this coheſion being a ſpecies of preſſive force, acting againſt the ſides of the wedge, it is abſurd to attempt to compare it with the percuſſive force of the mallet. For the momentum or percuſſive force of any body being as its quantity of matter multiplied into its velocity, it will be to its preſſive force, as that velocity to nothing, that is, will be indefinitely greater, and therefore the leaſt percuſſive force will overcome the greateſt preſſive force; conſequently there never can be an equilibrium between them. But though how great ſoever the preſſive force, and how ſmall ſoever the

percussive force, the body will necessarily be moved by the latter, at least for some short time; yet after the stroke is given, the pressive force may quickly prevail, and urge back the body, which the other force had driven forward. And this would frequently happen in the cleaving of wood, if the sides of the wedge were perfectly smooth. For the weight of the wedge, and the cohesive force of the particles of the wood, being homogeneous forces, may be in equilibrium; and if the weight of the wedge were not equivalent to that cohesive force, it would presently be forced back from the place to which the impulse of the mallet had driven it. It is chiefly the roughness of the sides of the wedge, and of the surface of the wood, in contact with them, that prevents the wedge from receding: it is that roughness too, and the bluntness of the edge, which sometimes prevent the wedge from being moved by the stroke of the mallet. See Landen on the Mechanic Powers.

LECTURE

LECTURE XVI.

1. IF a body act upon a perfectly hard and ſmooth plane, the effect produced upon the plane is in a direction perpendicular to its ſurface.

Hence the reaction of the plane is alſo in a direction perpendicular to its ſurface.

2. If a weight be ſuſtained upon an inclined plane by a power whoſe direction makes an angle with the plane, the power will be to the weight, as the ſine of the plane's elevation, to the coſine of the angle which the direction of the power makes with the plane.

Fig. 12. Let the weight *A* be ſuſtained on the inclined plane *CD*, by a force acting in the direction *AF*, and let *EA* be perpendicular to *CD*; there are now three forces acting on the body *A*, in the direction of the ſides of the triangle *AEF*, viz. the reaction of the plane in the direction

tion *EA*, the weight in the direction *FE* perpendicular to the horizon, and the sustaining force, in the direction *AF*. Therefore, in case of equilibrium, the sustaining force is to the weight as *AF* to *FE*.

3. If the direction of the power be parallel to the plane, the power will be to the weight, as the height of the plane to the length.

For in this case *AF* coincides with *AC*, and *FE* with *CE*.

4. If the direction of the power be parallel to the base, the power will be to the weight, as the height of the plane to the base.

For in this case *AF* coincides with *AH*, and *FE* with *HE*.

LECTURE XVII.

1. THERE will be an equilibrium in the ſcrew, when the power is to the reſiſtance, as the interval between two contiguous threads, in a direction parallel to the axis, to the circumference deſcribed by the power.

All the ſimple mechanic engines, except the ſcrew, can be made to act upon each other, without the intervention of any other machine of a different ſpecies. But in compoſition with other engines of different kinds, the ſcrew may be applied with great efficacy.

2 In a machine compounded of any number of mechanic engines, the power is to the weight in a ratio compounded of the ratios of the power to the weight in each.

In the theory of mechanics we ſuppoſe all planes perfectly ſmooth, levers to have no weight, cords to be perfectly pliable,

pliable, and all ſurfaces in contact to have no friction; the allowance to be made for the difference between theory and practice muſt be determined by experiment.

3. If a heavy wheel be ſo attached to a machine as to acquire a revolution on its axis during the motion of the engine, it will render the motion more eaſy and regular.

Becauſe the periphery of the wheel being of conſiderable weight, when by the continued action of the power it has been ſet in motion, its momentum becomes very conſiderable; and by its inertia it endeavours to preſerve that motion; conſequently ſmall variations in the intenſity of the moving power, or in the reſiſtance, will produce no ſenſible variation in the motion of the loaded wheel; neither of conſequence in the motion of the engine to which it is attached.

4. No force acting upon the parts of a machine and upon them only can give the whole a progreſſive motion.

5. Machines never leſſen the whole force neceſſary to perform any work, but are uſed either to diminiſh the force applied at once, by lengthening the time; or to ſhorten the time, by increaſing the force applied at once.

LECTURE

LECTURE XVIII.

1. IF a body move uniformly, the ſpace deſcribed may be repreſented by the area of a right-angled parallelogram, one of whoſe ſides repreſents the time of the body's motion, and the other its velocity.

In computations relating to forces, motions, times, velocities, or ſpaces deſcribed, we generally conſider only the proportions of thoſe quantities to each other, and not the abſolute quantities; in ſuch caſes our computations are much facilitated by ſubſtituting mathematical quantities inſtead of phyſical, provided they have the ſame proportions to each other; and then we may transfer our reaſoning on the mutual proportion of the exponents, to the mutual proportion of the phyſical quantities which they repreſent. Thus in uniform motions, the magnitude of the ſpaces deſcribed depends on the velocities with which the bodies move, and on the times of their motion, in the very ſame

manner that rectangles depend on their bases and altitudes; and hence, in our calculations respecting uniform motions, we may express the spaces described by rectangles, whose bases and altitudes have the same proportion to each other with the times and velocities.

2. Any force acting continually on a body in the same direction will produce a continual acceleration or retardation of its motion.

3. If the accelerative force be constant, the velocity generated or destroyed will be directly as the time; or the motion will be uniformly accelerated or retarded.

4. In uniformly accelerated or retarded motions, the spaces described may be represented by similar right-angled triangles, whose bases express the times, and altitudes the last acquired or initial velocities.

5. The spaces described by bodies uniformly accelerated in different times, are as the squares of the times, or of the last acquired velocities.

Hence, since a body, by the force of gravity, describes 193 inches in 1″, if T be the time of a body's fall through any space S, we have $1^2 : T^2 :: 193 : S = 193T^2$, in inches; and $T = \sqrt{\frac{S}{193}}$, in seconds.

6. If a body, projected with different initial velocities, be retarded by any given constant force until those velocities be destroyed, the spaces which the body

body defcribes, will be as the fquares of the initial velocities.

7. When bodies are impelled by any accelerating force through different fpaces, if thofe fpaces be as the fquares of the laft acquired velocities, the force of acceleration is conftant.

8. When a body, projected with different velocities, is retarded by any force until the velocity be deftroyed, if the whole fpaces be always as the fquares of the initial velocities, the retarding force is conftant.

Hence, fince the depth to which mufquet or cannon balls of a given diameter and weight, penetrate into planks of wood or banks of earth, are in a duplicate ratio of the initial velocities, the force by which the wood and earth refift, is conftant.

9. The fpaces defcribed by bodies uniformly accelerated from reft, in the feparate moments of time, are as the odd numbers, 1, 3, 5, 7, &c.

10. The fpace defcribed by a body uniformly accelerated is half the fpace it would defcribe, in the fame time, with the laft acquired velocity continued uniform.

Hence we derive an eafy formula for determining the velocity V acquired by a body falling from reft, by the conftant action of gravity, through any fpace S; let a be the velocity acquired in a fecond by a body falling through the fpace g, then $a=2g$, and $V : 2g :: \sqrt{S} : \sqrt{g}$, therefore

 V=

$V=\sqrt{4gS}$. Alſo $S=\frac{V^2}{4g}$; but $S=gT^2$, therefore $\frac{V^2}{4g}=gT^2$; whence $V=2gT$; thus ſince $g=193$ inches, when $T=1''$, if T be the time in ſeconds, then $V=2\times193\times T$, in inches; and $T=\frac{V}{2\times193}$, in ſeconds.

11. If moving forces communicate the ſame velocity in a given time to different bodies, they will be as the quantities of matter in the bodies moved.

Hence, ſince all bodies, whatſoever be their weights, deſcend near the earth's ſurface with equal velocities, the air's reſiſtance being removed, the force of gravity exerted on bodies is proportional to their quantities of matter.

12. The ſpaces deſcribed from reſt are, cæteris paribus, as the moving forces.

13. If a body fall freely by the action of any force F, compared with gravity repreſented by unity, then $S=gFT^2$; $V=\sqrt{4gFS}$, and $T=\frac{V}{2gF}$ $=\sqrt{\frac{S}{gF}}$.

14. If the force F be variable, it may be ſuppoſed conſtant during an indefinitely little time; and if $\dot{S}$ be the indefinitely little ſpace deſcribed in that time, and $\dot{V}$ the contemporaneous variation of the velocity, we ſhall have $V\dot{V}=2gF\dot{S}$.

For, by the laſt article $V^2 : v^2 :: FS : fs$, therefore $V^2= FS\times\frac{v^2}{fs}$; where v, f, s and F are conſtant quantities, and V and S variable; then taking the leaſt contemporary variations, $2\,V\dot{V}=F\dot{S}\times\frac{v^2}{fs}$; and making f the force of gravity $=1$,

$= 1$, $v = 2g$, and consequently $s = g$, we have $2V\dot{V} = 4gF\dot{S}$.

15. The spaces described from rest, by constant forces, are, cæteris paribus, inversely as the quantities of matter moved.

16. In general, the spaces described from rest are as the moving forces and the squares of the times directly, and the quantities of matter moved inversely.

17. If M represent the moving force, and Q the quantity of matter moved, then $F = \frac{M}{Q}$; hence $S = \frac{M}{Q} \times gT^2$, $V = \sqrt{\frac{4gMS}{Q}}$; and $T = \frac{VQ}{2gM}$.

18. If the moving force be inversely as the space described from rest, the quantity of matter will be inversely as the square of the velocity.

For $V^2Q = MS$, therefore if $M :: \frac{1}{S}$, then $Q :: \frac{1}{V^2}$.

19. The force with which a body descends along an inclined plane, is to the force of gravity, as the height of the plane to the length.

20. If two bodies descend from the highest point of an inclined plane at the same instant, one of them will fall through the perpendicular height, while the other descends on the plane to the intersection of a perpendicular drawn from the opposite angle.

Fig.

Fig. 13. From *C* the right angle let fall the perpendicular *CI* upon the plane *BD*; complete the parallelogram *ICBE*, the body at *B* is acted on by two forces, its gravity and the reaction of the plane, whose direction and magnitude are represented by the lines *BC*, *BE*; therefore by their joint action, it will describe the diagonal *BI* of that parallelogram, in the same time that it would describe *BC* by the separate action of gravity.

21. If the diameter of a circle be perpendicular to the horizon, and chords be drawn from either extremity, the times of descent down all the chords will be equal.

22. The velocity of a body descending along an inclined plane is uniformly accelerated.

23. The velocity acquired in descending along an inclined plane is equal to that which it would acquire in falling freely through the altitude.

24. The velocity acquired in descending along a system of inclined planes is equal to that which would be acquired in falling freely through the altitude of the system.

25. The velocity acquired by descending down the arch of any curve is the same with that which would be acquired in falling through its altitude.

26. The velocities acquired in descending through different arcs of the same circle, are as the chords of the arcs.

27. The

27. The time of a body's defcent along an inclined plane, is to the time in which it would fall freely through the altitude, as the length to the height.

28. The times of defcent along fimilar inclined planes, or fyftems of inclined planes, are as the fquare roots of the lengths.

29. The times of a body's defcent along fimilar arches, fimilarly fituated with refpect to the horizon, are as the fquare roots of the arches.

LECTURE

LECTURE XIX.

1. IF a pendulum vibrate in the arc of a circle, the velocity of the ball, at the loweſt point, will be as the chord of the arc, which it deſcribes in its deſcent.

2. The force which accelerates a pendulum, is to the force of gravity, as the ſine of its angular diſtance from the loweſt point to radius.

3. If the circular arc be increaſed, the time of vibration will alſo be increaſed.

If the force accelerating the pendulum increaſed in the ſame proportion with the arch to be deſcribed, the oſcillations would be iſochronal; but the accelerating force, in reality, increaſes as the right ſine of the arch; and the ſine of an arch increaſes in a leſs ratio than the arch itſelf; therefore the force which accelerates a pendulum increaſes in a leſs ratio than it ought in order to render the vibrations iſochronal

ifochronal; and of confequence, the time of vibration in a larger arch is longer than in a lefs arch.

4. The times of vibration in very fmall circular arcs are very nearly equal.

5. If a circle roll upon a right line, any point of its periphery will defcribe a curve called a cycloid.

6. If a circle be defcribed upon the axis of a cycloid, and an ordinate be drawn from the axis parallel to the bafe, the part of the ordinate intercepted between the circle and the cycloid will be equal to the arc of the circle intercepted between the vertex and the point where the ordinate meets it.

7. The chord of the above mentioned arc of the circle is parallel to a tangent to the cycloid at the point where the ordinate meets it.

8. The cycloidal arc intercepted between the vertex and the point where the ordinate meets it, is double of the chord of the correfponding circular arc.

9. If two equal femicycloids be joined at their bafe, and have their vertex downwards and axes vertical, and a pendulum equal in length to one of them be fufpended from the point where they touch, and vibrate between them, it will defcribe a cycloid.

10. If a pendulum defcribe an arc of a cycloid, its velocity at any point varies as the right fine of

a circular arc, whofe diameter is equal to the arc of the cycloid defcribed, and verfed fine equal to the fpace paffed over.

11. The accelerating force of a pendulum vibrating in a cycloid varies as the arc of its diftance from the loweft point.

12. The time in which a pendulum vibrates in a cycloid is to the time in which a body would defcend down the axis, as the circumference of a circle to its diameter.

13. All the vibrations in the fame cycloid are performed in the fame time.

14. Pendulums of the fame length vibrate in equal arcs in the fame time, whatever be their weights.

15. The time of an ofcillation in an indefinitely little circular arc, is to the time in which a body would fall through half the length of the pendulum, as the periphery of a circle to its diameter.

16. The time of defcent down an indefinitely little arc, is to the time of defcent along its chord as 3.1416 to 4, or as the periphery of a circle to four times the diameter.

17. The time of vibration, in any given latitude, is as the fquare root of the length of the pendulum.

For

For the time of vibration is to the time of defcent through half the length of the pendulum in a given ratio.

18. If a clock keep true time very nearly, the variation in the length of the pendulum neceffary to correct the error, will be equal to twice the product of the length of the pendulum and the error in time, divided by the time of obfervation.

Let T denote the time of vibration, L the length of the pendulum, and $\dot{T}$ the indefinitely little variation of time produced by $\dot{L}$ the indefinitely little variation in the length of the pendulum; then $T^2 : T^2 + 2T\dot{T} + \dot{T}^2 :: L : L + \dot{L}$; but $\dot{T}^2$ is indefinitely little with refpect to T, and therefore may be neglected, and dividing by T, we have $T : 2\dot{T} :: \dot{L} : L = \frac{2L \times \dot{T}}{T}$; but the error $\dot{T}$ in the time T, is to the error $\dot{t}$ in the time of obfervation t, as T to t; therefore $\dot{L} = 2L \times \frac{\dot{t}}{t}$, nearly.

19. In clocks whofe pendulum rods are metallic, and keep true time very nearly, the error in a given time, occafioned by any given change of temperature, will be the fame.

For t being given, $\dot{t} :: \frac{\dot{L}}{L}$; but $\dot{L}$, the variation in the length of a metallic bar, by the variation of temperature, is directly as L its length.

20. The fpace through which a body falls in the time of the vibration of any pendulum, is to

half the length of the pendulum, in the duplicate ratio of the periphery of a circle to its diameter.

A pendulum vibrating ſeconds is found to be 39.1 Engliſh inches long; therefore the ſpace deſcribed by a falling body in a ſecond is $= \frac{39.1 \times \overline{3.1416}|^2}{2} = 193,3$ inches.

The beſt method of aſcertaining the length of a pendulum is that which was firſt propoſed by Mr. Hatton, and afterwards executed by Mr. Whitehurſt. It conſiſts in the application of a moveable point of ſuſpenſion to the ſame pendulum; which thus gives the abſolute effect of two pendulums, the difference of whoſe lengths is known, being the interval between the points of ſuſpenſion in the two caſes; and the ratio of their lengths is alſo known from obſerving the number of vibrations performed in a given time. Whence, there being two equations and two unknown quantities, the actual lengths of the pendulums themſelves are eaſily deduced.

21. The times in which different pendulums vibrate in very ſmall arches, are to each other in a ratio compounded of the ſub-duplicate ratio of the lengths of the pendulums directly, and the ſub-duplicate ratio of the abſolute forces of gravity inverſely.

Hence, if the times be given, the forces of gravity are directly as the lengths of the pendulums.

22. The particles of matter which compoſe any body revolving on an axis, reſiſt, by their inertia, the

the communication of motion to any given point, with forces which are as the particles themſelves and the ſquares of their diſtances from the axis of motion jointly.

Fig. 14. Suppoſe a force F applied at any point D, in order to communicate motion to a ſyſtem of particles, each of which is P, revolving at determinate diſtances round the centre of motion S. Let D be a quantity of matter which, if concentrated in D, will have the ſame effect in reſiſting the communication of motion to the point D, by its inertia, when the particle P is removed, as the particle P revolving at the diſtance PS. The effect of the given force F, applied to the point D, to move a body at that point, is to its effect to move a body at P, inverſely as theſe diſtances, or as PS to DS; and if theſe bodies be moved with equal angular velocities, their diſtances from the axis will be as the ſpaces deſcribed in a given time; therefore the moving forces are inverſely as the ſpaces deſcribed; conſequently the quantities of matter muſt be inverſely as the ſquares of the velocities, Art. 18. Lect. 18. that is, inverſely as the ſquares of the diſtances from the axis; that is, $D : P : : PS^2 : DS^2$, and $D = \frac{P \times PS^2}{DS^2}$; that is, the reſiſtance of P at the diſtance PS, is equivalent to the reſiſtance of the maſs $\frac{P \times PS^2}{DS^2}$ at the diſtance DS; and the reſiſtance of all the particles, or of the whole revolving body is equal to the ſum of all the quantities $\frac{P \times PS^2}{DS^2}$.

23. The force which accelerates the point D of any

any body revolving on an axis, to which point that force F is applied, is equal to the product of the force into the ſquare of the diſtance DS, divided by the ſums of the products of all the particles into the ſquares of their reſpective diſtances from S the centre of motion.

For the maſs moved is the ſum of all the $\frac{P \times PS^2}{DS^2}$; and the moving force is F: but the accelerating force is had by dividing the moving force by the maſs, which therefore is the ſum of all the $\frac{F \times DS^2}{P \times PS^2}$.

24. The centre of oſcillation is that point in the axis of a vibrating body, in which if all the matter of the ſyſtem were collected, any force applied there would generate the ſame angular velocity in a given time, as the ſame force at the centre of gravity, the parts of the ſyſtem revolving in their reſpective places.

Hence the centre of oſcillation lies in a right line paſſing through the centre of gravity, and perpendicular to the axis of motion.

25. If the ſum of the products of each particle of a pendulum multiplied into the ſquare of its diſtance from the centre of motion, be divided by the weight of the whole multiplied into the diſtance of the centre of gravity from the axis of motion, the quotient will be the diſtance of the centre of oſcillation from the ſame point.

The

The motive force of the pendulum is its weight multiplied into the fine of the angular diftance of the centre of gravity from the loweft point, and divided by radius; this force, which call F, acts at the centre of gravity. The force which accelerates the centre of gravity D is the fum of all the $\frac{F\times DS^2}{P\times PS^2}$, and therefore the force which accelerates O the centre of ofcillation is the fum of all the $\frac{F\times DS\times OS}{P\times PS^2}$, the accelerations being directly as the diftances of thefe centres from the axis of motion. Now if M, the fum of the particles, were collected in O, and the given force applied there, the force accelerating O would be $=\frac{F}{M}$. But the force accelerating O, in this cafe, muft be equal to the force accelerating O, when F is applied at the centre of gravity, that is $\frac{F}{M}=$the fum of all the $\frac{F\times DS\times OS}{P\times PS^2}$ therefore $OS=$the fum of all the $\frac{P\times PS^2}{M\times DS}$.

26. In a compound pendulum, confifting of feveral bodies revolving round a common axis, the centre of ofcillation is thus determined: add together the feveral products of the weights of each body into the diftances of the refpective centres of gravity and ofcillation from the common centre of motion, and divide the fum by the product of the whole fyftem into the diftance of the common centre of gravity from the axis of motion; the quotient

quotient will be the diſtance of the centre of oſcillation from the ſame axis.

From the laſt article it appears, that the ſum of all the products, in each part of the compound pendulum, which are formed by multiplying each particle into the ſquare of its diſtance from the axis of motion, is equal to the product of the diſtances of the centres of oſcillation and gravity of that part from the ſame axis into the whole weight of that part; therefore the ſum of all the $P \times PS^2$ in the whole pendulum is equal to the ſum of all the products in each part of the pendulum, which are formed by multiplying the weight of each part into the product of the diſtances of the centres of oſcillation and gravity of that part from the common axis of motion; therefore if this latter ſum be divided by the product of the weight of the compound pendulum into the diſtance of the common centre of gravity from the axis of motion, the quotient, by the laſt article, will be the diſtance of the centre of oſcillation from the ſame axis.

27. The centre of gyration is a point, in which if all the matter contained in a revolving ſyſtem were collected, any point to which a given force is applied to communicate motion, would be accelerated in the ſame manner, as when the parts of the ſyſtem revolve in their reſpective places; and conſequently the angular velocity generated in a given time, in both caſes, is the ſame.

28. If

28. If P denote each particle of which a body is composed revolving on the axis S, the distance of the centre of gyration R from that axis will be equal to the square root of the sum of all the $\frac{P \times PS^2}{M}$.

Let any force F be applied to move the body, at any distance SD from the axis. If the force which accelerates a given point be the same in any two cases, the absolute velocity of that point, generated in a given time, must be the same; and consequently the angular velocity of the body will be equal in both cases. The force which accelerates the point D is $=$ the sum of all the $\frac{F \times DS^2}{P \times PS^2}$; now let M be concentrated in R, the centre of gyration, and the force which accelerates D will be $= \frac{F \times DS^2}{M \times SR^2}$. These forces are equal by the hypothesis; that is, the sum of all $\frac{F \times DS^2}{P \times PS^2} = \frac{F \times DS^2}{M \times SR^2}$; therefore $SR^2 =$ sum of all the $\frac{P \times PS^2}{M}$; and SR equal to the square root of that quantity.

Thus if a slender rod, whose length $SP = a$, revolve on the point S, the distance of the centre of gyration from that point will be $= a\sqrt{\frac{1}{3}}$: for let $SD = x$; then the sum of all the $\frac{PS^2 \times P}{M} =$ the fluent of $\frac{x^2\dot{x}}{a} = \frac{x^3}{3a} = \frac{1}{3}a^2$, when $x = a$.

Fig. 27. If a circle or cylinder EDB revolve on its centre or axis S, and the radius $SD = r$, the diſtance of the centre of gyration from the centre or axis will be $= r\sqrt{\frac{1}{2}}$: for let $SA = x$, and $p =$ the periphery of a circle whoſe diameter is unity; then the circumference of the annulus whoſe radius is $x = 2px$, and its area $= 2px\dot{x}$; and the area of the circle $EDB = pr^2$; therefore the ſum of all the $\frac{PS^2 \times P}{M} =$ the fluent of $\frac{x^2 \times 2px\dot{x}}{pr^2} = \frac{x^4}{2r^2} = \frac{1}{2}r^2$, when $x = r$.

In the ſame manner, if a globe, whoſe radius is r, revolve about one of its diameters, the diſtance of the centre of gyration from the centre will be $= r\sqrt{\frac{2}{5}}$.

29. The diſtance of the centre of gyration from the axis of motion is a mean proportional between the diſtances of the centres of oſcillation and gravity from the ſame axis.

Fig. 14. For $OS =$ the ſum of all the $\frac{P \times PS^2}{M \times DS}$, therefore $OS \times DS =$ ſum of all the $\frac{P \times PS^2}{M} = SR^2$.

When the axis of motion paſſes through the centre of gravity, the above centre is called *The Principal centre of Gyration.*

30. The diſtance between the centre of gravity and principal centre of gyration is a mean proportional

tional between the diſtances of the centres of motion and oſcillation from the centre of gravity.

For $PS^2 = PD^2 + DS^2 \pm 2PDS$, therefore $OS = \int. \frac{P \times \overline{PD^2 + DS^2 \pm 2PDS}}{M \times DS}$. But from the nature of the centre of gravity, $\int. P \times PD$, on each ſide of D, are equal, and becauſe $2DS$ is conſtant, the ſum of all the $\pm 2PDS = 0$. Moreover, $\int. P \times DS^2 = M \times DS^2$, therefore $OS = DS + \int. \frac{P \times PD^2}{M \times DS}$, and $OD = \int. \frac{P \times PD^2}{M \times DS}$; therefore $\int. \frac{P \times PD^2}{M}$, that is the diſtance between the centre of gravity and principal centre of gyration, is a mean proportional between OD and DS.

31. If the centre of oſcillation be made the point of ſuſpenſion, the point of ſuſpenſion will become the centre of oſcillation; the plane of vibration being ſuppoſed to continue the ſame.

When S is the point of ſuſpenſion, the diſtance of the centre of oſcillation from the centre of gravity $= OD = \int. \frac{P \times PD^2}{M \times DS}$, as appears from the demonſtration of the laſt article; and when O is the point of ſuſpenſion, x the diſtance of the centre of oſcillation from the centre of gravity $= \int. \frac{P \times PD^2}{M \times DO}$, therefore $x = DS$, and the diſtance of the centre of oſcillation from $O = SO$.

32. The time of vibration will be the leaſt poſſible, when the axis of motion paſſes through the principal centre of gyration.

Let the diſtances of the axis of ſuſpenſion, the principal centre of gyration, and centre of oſcillation from the centre of gravity, reſpectively, be x, r, and y; then $xy = r^2$ a conſtant quantity; therefore $x+y$, the length of the pendulum, is leaſt when $x = y = r$, by prop. 4. Elem. 2.

33. When a pendulum is at reſt, if a body impinge on it in an horizontal direction, the ſame velocity will be communicated to the point of impact, as if the maſs of the pendulum were removed, and inſtead of it an equivalent maſs were concentrated in the point of impact, the quantity of the equivalent maſs being to that of the pendulum, in a duplicate ratio of the diſtances of the centre of gyration, and the point of impact from the axis of motion.

Fig. 15. Let *ABCD* repreſent a pendulum, whoſe axis of motion is *S*, and let any impact be impreſſed on the point *F*, in an horizontal direction, and perpendicular to the vertical plane *ABCD*, the pendulum being at reſt; alſo let Q be the quantity of matter which being concentrated in *F*, the ſame angular velocity will be communicated by the impact, as when *W* the weight of the body is concentrated in *R* the centre of gyration. Since the particles of matter which compoſe the pendulum reſiſt, by their inertia, the communication of motion with forces which are as the particles themſelves multiplied into the ſquares of their diſtances from the axis of motion, and ſince the reſiſtances in the two caſes muſt be equal, we have $W \times SR^2 = Q \times SF^2$, and $Q = W \times \frac{SR^2}{SF^2}$.

Hence

Hence V the velocity of the impinging body M may be determined; let v be the velocity communicated to the point of impact; then, by the laws of the collision of non-elastic bodies, $M : M+Q :: v : V = v \times \frac{M+Q}{M}$. To determine v, measure the arc described by O the centre of oscillation in its ascent after impact: let its versed sine be r; then the velocity of the centre of oscillation at its lowest point $= \sqrt{4gr}$, by Art. 10 and 25, Lect. 18. and the velocity of the point of impact $= \frac{SF}{SO} \times \sqrt{4gr}$.

34. If motion be communicated by a hanging weight to a system revolving on a fixed axis passing through the centre of gravity, and the moving force act always at a given distance from the axis of motion, it will generate in the revolving system the same motion, as it would acquire in the same time by falling freely by its gravity from a state of rest.

Fig. 16. Let ABC represent a body moveable round its centre of gravity S, thro' which an horizontal axis of motion passes; let R be the centre of gyration, p the weight of the body which gives motion to the system, by means of a line DP wound round the circle DEF, and w the weight of the system. Let the distance of the centre of gyration from the axis $SR = r$, $SD = d$; the inertia of the whole system is equivalent to the weight $\frac{wr^2}{d^2}$ uniformly diffused through the periphery DEF, every point of which moves

with

with the ſame velocity with p; the moving force is p, therefore the accelerating force is $\frac{pd^2}{wr^2}$, p being ſuppoſed deſtitute of inertia, or incomparably leſs than the weight of the ſyſtem: then, in the time t, the velocity generated in the point D or p will be $= \frac{2gtpd^2}{wr^2}$; which multiplied into the quantity of matter $\frac{wr^2}{d^2}$, gives $2gtp$ for the whole quantity of motion generated.

Again, if p were to deſcend by the force of gravity unimpeded, it would acquire, in the time t, the velocity $2gt$; which multiplied into the quantity of matter p, would give $2gtp$, the ſame quantity of motion as before.

Hence it follows, that the permanency of motion, eſtimated by the product of the quantity of matter and velocity, obtains in bodies which revolve on fixed axes.

LECTURE

LECTURE XX.

1. THE centre of Percuſſion is that point in a body revolving about an axis, at which if it ſtruck an immoveable obſtacle, all its motion would be deſtroyed, or it would incline neither way.

Fig. 17. If during the vibration of a ſyſtem of bodies round a fixed axis, ſuch an obſtacle be oppoſed to any point *O*, as entirely to deſtroy the motion of that point, every other particle of the ſyſtem will endeavour, by its inertia, to proceed in the direction of its motion, that is, of the tangent of the circular arc it was deſcribing, the inſtant that *O* was ſtopped. Theſe forces will therefore act on the ſyſtem, to turn it round *O*; and if the ſum of the forces on each ſide of *O* ſhould be unequal, the motion of the ſyſtem will not be deſtroyed, when *O* is ſtopped: but ſince the forces which

which act on the pendulum between O and S, have an effect to continue the motion of the fyftem, contrary to thofe which are impreffed on the other fide of O, if the point O be fo fituated, that the fum of the forces to turn the fyftem round O, on each fide of that point, may be exactly equal, the inftant that O is ftopped, the whole motion of the fyftem will be deftroyed.

2. If a pendulum vibrating with a given angular velocity ftrike an obftacle, the effect of the impact will be the greateft, when it is made at the centre of percuffion.

For in this cafe, the obftacle receives the whole revolving motion of the pendulum; whereas if the blow be ftruck in any other point, a part of the pendulum's motion will be employed in endeavouring to continue its rotation.

3. If a body revolving on an axis, ftrike an immoveable obftacle at the centre of percuffion, the point of fufpenfion will not be affected by the ftroke.

4. The diftance of the centre of percuffion from the axis of motion is equal to the diftance of the centre of ofcillation from the fame.

Fig. 17. Let S be the axis of motion, G the common centre of gravity, and P one of the particles compofing the fyftem; draw $SGOL$, fince the angular motion of all the particles is the fame, the abfolute velocity will be proportional to the diftance from the axis of motion; and if in the diftance 1 the velocity be expreffed by unity, the velocity of P will be

be $= PS$, and the quantity of motion $= P \times PS$, which will act in the direction PR perpendicular to SP; produce PR to D, and let fall on it the perpendicular OD from O; then will $P \times PS \times OD$ be the force of the particle P to move the system round O. But, because of similar triangles, $DO = \frac{PS \times RO}{RS} = PS \times \frac{SO - RS}{RS}$; and if PA be perpendicular to OS, we shall have $SA = \frac{PS^2}{RS}$. Therefore the same entire force $= P \times PS^2 \times \frac{\overline{SO - RS}}{RS} = P \times SA \times SO - P \times PS^2$. But since O is the centre of percussion, the sum of all the $P \times SA \times SO =$ the sum of all the $P \times PS^2$; therefore $SO =$ sum of $\frac{\text{all the } P \times PS^2}{\text{all the } P \times SA} =$ the sum of all the $\frac{P \times PS^2}{M \times GS}$.

This demonstration supposes, that the centre of percussion is required in a plane passing through the axis of motion and centre of gravity. If it be required in any other plane as So, passing through the axis of motion, from G the centre of gravity let fall the perpendicular Gg on So; and by the same argument as before, $So =$ the sum of $\frac{\text{all the } P \times PS^2}{\text{all the } P \times Sa} =$ the sum of all the $\frac{P \times PS^2}{M \times gS}$; and $So : SO :: GS : gS$, and the angle $oOS =$ the angle GgS, which therefore is a right angle.

Hence it follows, that a body has various centres of percussion, according to the plane passing through the axis of motion in which the impact is made; and the right line oO is their Locus.

5. The centre of percussion does not always lie in a right line perpendicular to the axis of motion and passing through the centre of gravity.

Fig. 18. Let the triangle ACB vibrate on the axis $mSCn$ parallel to the base AB; let G be the centre of gravity, $SGE'E$ perpendicular to AB, and $AxD'E'zB'$ parallel to the same, and therefore parallel to the axis. Let the perpendicular $CL = p$, $AB = a$, $DL = d$, and $SE' = y$; then $GE' = y - \frac{2}{3}p$, and therefore $D'E' = \frac{d}{p} \times \overline{y - \frac{2}{3}p}$. The sum of all the $x \times mx \times mS$ in the line $A'E' = mx \times \frac{A'E'^2}{2}$ of all the $z \times nz \times Cn$, in $E'B' = nz \times \frac{E'B'^2}{2}$; therefore the difference or efficacy of the line $A'E'B'$ to turn the plane about $SE = SE' \times \frac{A'E'^2 - E'B'^2}{2} = \frac{1}{2} SE' \times A'B' \times 2D'E'$ Whence the fluxion of the sum of the efficacy $= SE' \times A'B' \times D'E' \times$ flux. of $SE' = y\dot{y} \times \frac{ay}{p} \times \frac{d}{p} \times \overline{y - \frac{2}{3}p}$. The fluent, which has no correction, $= \frac{ady^4}{4p^2} - \frac{2ady^3}{9p}$, which when $y = p$, becomes $\frac{adp^2}{36}$; therefore the triangle is not balanced on SE, except when $d = o$, or the triangle is Isosceles.

To find the distance of the centre of percussion from the plane passing through the centre of gravity, and perpendicular to the axis of motion; See Hydrostatics, Lect. 2.

6. The centre of spontaneous rotation is that point which remains at rest, the instant the body is struck, or about which the body begins to revolve.

7. The

7. The centre of ſpontaneous rotation is the ſame with the centre of ſuſpenſion correſponding to the centre of percuſſion, the centre of percuſſion being that point where the body is ſtruck.

For the action of the body againſt an immoveable obſtacle in the centre of percuſſion, muſt have the ſame effect upon the body, as if the body had been at reſt, and the obſtacle had ſtruck the body; in which latter caſe, the centre of ſuſpenſion would not be affected, and therefore it becomes the centre of ſpontaneous rotation.

8. If an impact be made on any point of the axis of a regular body, and that point be conſidered as the point of ſuſpenſion, the correſponding centre of oſcillation will be the centre of ſpontaneous rotation.

Becauſe if the centre of oſcillation be made the point of ſuſpenſion, the point of ſuſpenſion will become the centre of oſcillation; and the centre of oſcillation is the ſame with the centre of percuſſion, in the axis of a regular body.

9. The poſition of the centre of ſpontaneous rotation does not depend on the magnitude of the impact.

10. The rotation round the centre of ſpontaneous converſion is cauſed by the different velocities communicated to the different points of the ſyſtem.

Fig. 19. Let the inflexible rod *RS*, whoſe centre of gravity is *G*, be ſtruck in the point *F*, and in the direction *DF* perpendicular to *RS*; ſince the impact is direct, it cannot be reſolved, but muſt be entirely communicated to the particles, endeavouring to move them in directions parallel to *DF*. Now when the impact is made on *F*, in the direction

DF, the particles will refift in the contrary direction, and the point of impact will become a fulcrum, the particle *S* acting by the lever *SF*, and the particle *R* by the lever *RF*; hence if thefe levers be unequal, the refiftances will be fo likewife, and fince the refiftance on the fhorter lever *FS* is lefs, that extremity of the rod will move fafter than the extremity *R*. But the points *S*, *G*, *R* muft begin to move in the direction of the impelling force, that is, in a direction parallel to *DF*; and becaufe the determination of motion once impreffed on the centre of gravity, is not altered, except by the impulfe of external force acting in fome other direction, it follows that the direction in which the centre of gravity *G* proceeds from the very beginning of its motion, will be perpendicular to the line *RS*; and the point *S* moving fafter than *R*, the rod at the fame time will revolve round fome point, which is called the centre of Spontaneous Converfion.

11. If a right line be drawn through the centre of gravity of a body, to whatever point of this line the impact be applied, the velocity of the centre of gravity will be the fame.

Fig. 20. To whatever point of QP the fame force F is applied, the incipient motion $Q \times Qq + P \times Pp$ of the two particles Q, P, connected by the inflexible line QP, will be the fame, by the fecond law of motion; and confequently, the velocity of the centre of gravity, or Gg, is always the fame as if both particles were placed at G, and acted on by the fame force F; for $Gg = \frac{P \times Pp + Q \times Qq}{P + Q}$, by the nature of the centre of gravity.

Hence

Hence the permanency of the fame quantity of motion obtains in rotations round a centre of Spontaneous Gyration.

12. If the impact be made in a direction paffing through the centre of the impelling body, the velocity of the centre of gravity of the body ftruck will be equal to the product of the quantity of motion of the impelling body into the diftance between the centres of gravity and fpontaneous rotation, divided by the fums of the products of the impelling body into the diftance of the point of impact from the centre of rotation, and of the impelled body into the diftance between the centres of rotation and gravity.

Let the quantity of matter of the impinging body be D, its velocity V, B the quantity of matter of PQ the body which is ftruck, in the direction DF paffing through the centre of D, g the velocity of the centre of gravity, and S the centre of fpontaneous converfion of PQ; then $SG : SF : g :$ the velocity of F, which is therefore $= \frac{SF \times g}{SG}$; therefore $V - \frac{SF \times g}{SG} =$ the velocity loft by D, in the direction DF; and (3^d law of motion) $D \times \overline{V - \frac{SF \times g}{SG}}$ $= B \times g$, and $D \times \frac{\overline{V \times SG - SF \times g}}{SG} = Bg$, and $g = \frac{D \times V \times SG}{B \times SG + D \times SF}$.

If

If D be very ſmall with reſpect to B, the velocity of the centre of gravity will be $\doteq \frac{DV}{B}$.

13. The angular velocity of the centre of the ſyſtem round the centre of gravity, when the impelling body is indefinitely leſs than the ſyſtem, is equal to the momentum of the impelling body divided by twice the product of the maſs of the impelled body and the diſtance between the centres of gravity and ſpontaneous gyration into the periphery of a circle whoſe diameter is unity.

If a fixed axis paſſed through S, the centre of gravity would deſcribe a circle whoſe radius is SG, with the velocity $\frac{DV}{B}$. But the centre of gravity, not being fixed, will go on in the direction of its firſt impulſe; and if no rotatory motion had been communicated to the ſyſtem, the line PQ would have moved into the poſition xz, parallel to PS; but the angular motion about $S = PSp = xgp$, the contemporary angular velocity round the centre of gravity; the motion therefore of the ſyſtem will be compounded of the uniform rectilinear motion of the centre of gravity, in the direction Gg, perpendicular to RS, and the angular motion generated round the centre of gravity. Now ſince the periphery of a circle whoſe radius is $SG = 2p \times SG$, p being the periphery of a circle whoſe diameter is unity, we have this analogy, $\frac{DV}{B} : 2p \times SG ::$ one ſecond : the time of one revolution in ſeconds $= \frac{2p \times B \times SG}{DV}$

and

and consequently, the number of revolutions or parts of a revolution in a second, or the angular velocity $= \frac{DV}{2p \times SG \times B}$.

14. The centre of spontaneous conversion, during the motion of the system, describes the common cycloid.

For the motion of any point in the system is compounded of the uniform rectilineal motion of the centre of gravity, and of the angular motion generated round that centre; but the velocity with which the centre of spontaneous conversion would move round the centre of gravity, if there existed a rotatory motion only in the system, would be equal to that, with which the centre of gravity would move round it, if the centre of spontaneous conversion were fixed; since therefore the centre of spontaneous conversion has both a rotatory and progressive motion, each of which is equal to that of the centre of gravity, it will describe a cycloid.

LECTURE

LECTURE XXI.

1. CLEPSYDRÆ are inadequate to the accurate menſuration of time.

2. Clockwork, regulated by a ſimple balance, is inadequate to the accurate menſuration of time.

3. Clockwork, regulated by a pendulum vibrating in the arch of a circle, is inadequate to the accurate menſuration of time.

1ſt. Becauſe the vibrations in greater and ſmaller arches are not performed in equal times. 2dly. Becauſe the length of the pendulum is varied by heat and cold.

4. Clockwork, regulated by a pendulum vibrating in the arch of a cycloid, is inadequate to the accurate menſuration of time.

The

The isochronism of the vibrations of a cycloidal pendulum in greater and smaller arches, is true only on the hypothesis, that the pendulum moves in a non-resisting medium, and that the whole mass of the pendulum is concentrated in a point, both of which positions are false. For these reasons the application of the cycloid in practice has been entirely relinquished.

5. Modern time-keepers owe almost the whole of their superiority over those formerly made to two things. 1st. The application of a thermometer. 2dly. The particular construction of the escapement.

6. Metals expand by heat and contract by cold.

This is proved experimentally by the pyrometer. Metallic bars of the same kind are found to expand in proportion to their length. Metals of different kinds expand in different proportions; thus the expansion of iron and steel are as 3, copper 4½, brass 5, tin 6, lead 7. Hence pendulum rods, expanding and contracting by the successive changes of temperature, affect the going of the clocks to which they are applied.

Various have been the contrivances to correct the errors of pendulums from their contraction and expansion by heat and cold; which however may be reduced to four general classes, the mercurial, gridiron, lever, and gibbet pendulums.

7. If a metallic pendulum rod be attached to a hollow glass cylinder, partly filled with mercury, instead of a bob, the variation of the length of the

pendulum produced by the expanſion or contraction of the rod, will be corrected by the expanſion or contraction of the mercurial column contained in the glaſs cylinder.

The defect of this thermometer ſeems to ariſe from the expanſion of the mercury commencing ſooner than the expanſion of the rod.

8. If a pendulum be compoſed of an odd number of bars of two different metals alternately diſpoſed, ſo connected that the ball, which is attached to the middle bar, ſhall be carried down by the expanſion of every alternate bar beginning from the extremes, and upwards by the intermediate bars; then will the expanſions of all the bars together correct each other, provided the length of the ſum of all the alternate bars from the middle to the extreme bar on either ſide incluſive, be to the lengths of the intermediate bars, inverſely as the expanſions of the metals of which they are compoſed.

The principal objections to this mode of compenſation are, 1ſt. The difficulty of exactly adjuſting the lengths of the rods. 2dly. Of proportioning their thickneſs ſo, that they ſhall all begin to expand or contract at the ſame inſtant. 3dly. The connecting bars of a pendulum thus conſtructed are apt to move by ſtarts.

9. If a pendulum rod be compoſed of two coincident bars of different metals, attached to each other

other only at their upper extremities, that which expands leaſt bearing the fulcrums of two levers, which on their outer extremities ſuſtain the ball of the pendulum, while the bar which expands moſt, bears againſt both the other ends of the levers in the middle; the expanſions of theſe two bars will counteract each other, and preſerve the length of the pendulum unvaried.

10. If a bar of the ſame metal, form, and ſize with the pendulum rod be ſupported on a firm bracket at the lower end, and the other carry a tranſverſe piece attached to a ſpring at the upper extremity of the pendulum, which ſpring may be moved upwards and downwards between two cheeks, and thus determine the length of the pendulum, the expanſions of this bar and of the pendulum rod will correct each other.

It is however to be remarked, that the ſuſpenſion of a pendulum by a ſpring, is not ſo good as by an edge. 1ſt. Becauſe the reſiſtance in the former manner is greater than in the latter. 2dly. The centre of the motion of the pendulum varies according as the elaſticity of the ſpring changes. 3dly. When the ſpring has been lengthened by heat, the weight of the ball will prevent the cold from ſhortening it as much, ſo that the ſpring will be continually lengthening.

LECTURE XXII.

1. THE balance of a watch is analogous to the pendulum in its properties and uſe.

The ſimple balance is a circular annulus, equally heavy in all its parts, and concentrical with the pivots of the axis on which it is mounted. This balance is moved by a ſpiral ſpring called the balance-ſpring, the invention of the ingenious Mr. Hook.

2. The pendulum requires a leſs maintaining power than the balance.

Hence the natural iſochroniſm of the pendulum is leſs diſturbed by the relatively ſmall inequalities of the maintaining power.

3. The ſpring's elaſtic force which impels the circumference of the balance, is directly as the tenſion of the ſpring; that is, the weights neceſſary to counterpoiſe a ſpiral ſpring's elaſtic force, when the

the balance is wound to different diftances from the quiefcent point, are in the direct ratio of the arcs through which it is wound.

4. The vibrations of a balance whether through great or fmall arches are performed in the fame time.

For the accelerating force is directly as the diftance from the point of quiefcence; hence therefore the motion of the balance is analogous to that of a pendulum vibrating in cycloidal arches.

5. The time of the vibration of a balance is the fame as if a quantity of matter, whofe inertia is equal to that by which the mafs contained in the balance oppofes the communication of motion to the circumference, defcribed a cycloid whofe length is equal to the arc of vibration defcribed by the circumference, the accelerating force being equal to that of the balance.

Becaufe in both cafes the fpaces defcribed would be equal, as alfo the accelerating forces in correfponding points, and therefore the times of defcription.

6. If 1 denote the accelerating force of gravity, L the length of a pendulum vibrating feconds in a cycloid, a the femi-arc of vibration of the balance, T the time of vibration, and F the accelerating force of the balance; then will $T = \sqrt{\frac{a}{L \times F}}$.

For

For since the time of vibration of the balance is analogous to that of a pendulum in a cycloid, and the semi-arc of vibration is $=$ the length of the pendulum, $1'' : T :: \sqrt{L} : \sqrt{\frac{a}{F}}$, and therefore $T = \sqrt{\frac{a}{L.F}}$ in seconds.

7. Let g be the space which a body falling freely from a state of rest describes in $1''$, and $p = 3.14159$ the circumference of a circle whose diameter is unity, then will $T = \sqrt{\frac{p^2 a}{2gF}}$.

Because $L = \frac{2g}{p^2}$. This is the formula delivered by Mr. Atwood, page 10, of his Investigation of the times of vibration of a Watch Balance.

In this expression for the time of vibration, the letter a denotes the length of the semi-arc of vibration; if this arc should be expressed by a number of degrees $c°$, and r be the radius of the balance, then a will be $= \frac{prc°}{180°}$; and this quantity being substituted for a, the time of a vibration will be $T = \sqrt{\frac{p^3 rc°}{2gF \times 180°}}$; let the given arc be $90°$, in this case $T = \sqrt{\frac{p^3 r}{4gF}}$.

8. If the spring's elastic force, when wound through the given angle or arc $a = 90°$ from the quiescent position, be $= P$; the weight of the balance, and the parts which vibrate with it $= W$, the distance of

of the centre of gyration from the axis of motion $= \gamma$, then will $T = \sqrt{\frac{Wp^3\gamma^4}{4Prg}}$.

For the resistance of inertia by which the mass contained in the balance opposes the communication of motion to the circumference will be $\frac{W\gamma^2}{r^2}$; and consequently the force which accelerates the circumference at the angular distance a from the quiescent position is $\frac{Pr^2}{W\gamma^2} = F$, which being substituted in the former equation, we have $T = \sqrt{\frac{Wp^3\gamma^2}{4Prg}}$ which is the formula delivered by Mr. Atwood in his masterly essay above referred to.

These are expressions for the time of a vibration, whatever may be the figure of the balance, the other conditions remaining the same as above stated. If the balance be an annulus or a cylindrical plate, $\gamma = \frac{r}{\sqrt{2}}$, Art. 28. Lect. 19, and the time of vibration $T = \sqrt{\frac{Wp^3r}{8Pg}}$.

9. The times of vibration of different balances are in a ratio compounded of the direct subduplicate ratios of their weights and semidiameters, and the inverse subduplicate ratio of the tensions of the springs, or of the weights which counterpoise them, when wound through a given angle.

10. The times of vibration of different balances are in a ratio compounded of the direct simple ratio

ratio of the radii, and direct subduplicate ratio of their weights, and the inverse subduplicate ratio of the absolute forces of the springs at a given tension.

For the weights necessary to wind the spring through a given angle, are directly as the absolute forces of the springs, and inversely as the lengths of the levers at which they act, that is, directly as the absolute forces of the springs at a given tension, and inversely as the radius of the balance; let A be the absolute force of the spring at a given tension, then $T :: \sqrt{\frac{Wr}{P}}$; but $P = \frac{A}{r}$, therefore $T :: \sqrt{\frac{W.r^2}{A}}$; which is the formula discovered by Mr. Harrison, and delivered by Mr. Ludlam.

11. Hence the absolute force of the balance spring, the diameter and weight of the balance being the same, is inversely as the square of the time of one vibration.

12. The absolute force or strength of the balance spring, the time of one vibration and the weight of the balance being the same, is as the square of the diameter of the balance.

13. The weight of the balance, the strength of the spring and time of vibration being the same, is inversely as the square of the diameter.

Hence a large balance vibrating in the same time, with the same spring, will be much lighter than a small one.

14. If

14. If the rim of the balance be always of the ſame breadth and thickneſs, ſo that the weight ſhall be as the radius, the ſtrength of the ſpring muſt be as the cube of the diameter of the balance, that the time of vibration may continue the ſame.

For $T^2 :: \frac{W \times r^2}{A}$, but $W :: r$, therefore $T^2 :: \frac{r^3}{A}$; and T^2 being given, $A :: r^3$.

15. If a balance be made with two balls joined by a rod, and the weights and diſtances of theſe balls from their common centre of motion be unequal, but ſuch that each ſeparately would vibrate in the ſame time; the centre of gravity of theſe balls will not coincide with their centre of motion, nor will they poiſe each other.

For to make this equilibrium, W ſhould be inverſely as r; whereas to preſerve the times of their vibration the ſame, W muſt be inverſely as r^2.

16. The momentum of the balance is increaſed better by increaſing its diameter than its weight.

It is better to increaſe the momentum by increaſing the velocity than the weight, becauſe the velocity does not increaſe the friction as much as the weight. Now the number of vibrations being given, the velocity may be increaſed either by increaſing the arc of vibration, or increaſing the diameter of the balance; the latter method is preferable, becauſe the velocity is thus augmented,

without increafing the friction, which is in proportion to the fpace defcribed by the pivot, the weight of the balance being given.

17. A ftronger balance-fpring is preferable to a weaker.

Becaufe the force of this fpring upon the balance remaining the fame, whilft the difturbing force varies, the errors arifing from the variation will be lefs, as the fixed force is greater.

18. The longer a detached balance continues its motion the better.

Becaufe 1ft. The friction in this cafe is lefs, and therefore the natural ifochronifm of the vibration is lefs difturbed. 2dly. When applied to the watch, it requires a lefs maintaining power, and therefore the variations in the intenfity of the maintaining power will be lefs. 3dly. The maintaining power being lefs, the friction of the wheelwork will be lefs, and therefore the motion more regular. 4thly. The preffure on the efcapement will be lefs, and therefore the ofcillations of the balance lefs difturbed.

19. The greater is the number of vibrations performed by a balance in a given time, the lefs fufceptible is it of external agitations.

20. Slow vibrations are preferable to quick vibrations: but there is a limit; for if the vibrations be too flow, the watch will be liable to ftop.

If we regarded only the effect of external agitations, balances that vibrate quick fhould be preferred to fuch as vibrate flow; but they are attended with two inconvenien-

cies,

cies, greater than that which we would avoid: 1ſt. In two balances of the ſame weight and diameter, the friction on the pivots increaſes with the number of vibrations. 2dly. It appears by experience that the motion of the ſame detached balance continues longer, when its vibrations are ſlow, than when they are quick.

21. A balance ſhould deſcribe as large arches as poſſible, as ſuppoſe 240°, 260°, 300°, or an entire circle.

Firſt, becauſe the momentum of the balance is thus increaſed; and therefore the inequalities in the force of the maintaining power bear a leſs proportion to it, and of conſequence will have leſs influence. 2dly. The balance is leſs ſuſceptible of external agitations. 3dly. A given variation in the extent of the vibrations produces a leſs variation in the going of the machine.

But care muſt be taken, that in theſe great vibrations, the ſpring ſhall neither touch any obſtacle, nor its ſpires touch each other in contracting.

22. The times of vibration in larger arches are ſometimes ſhorter, ſometimes longer than in leſs arches.

Mr. Harriſon was the firſt who aſſerted, that the vibrations in larger arches are performed in leſs time than in ſhorter arches, in contradiction to the opinion generally held by philoſophers and workmen; which aſſertion appears ſometimes to be true. But it appears by experiment, that the elaſtic forces of ſpiral ſprings deviate from the

ifochronal law of variation in fome cafes by falling fhort of it, as well as in others by exceeding it.

23. A uniform fpiral fpring may be rendered perfectly ifochronal by adjufting its length and number of fpires.

This is the opinion of Mr. Berthoud; his reafoning feems to be this: if the fpring forming a fpiral of a certain fpecies be fo difpofed, that when wound through different angles, the accelerating elaftic forces of the fpires, from the centre towards the circumference, increafe fafter than they ought to do in order to render the vibrations ifochronal, it may be otherwife fo difpofed, namely by making the fpires approach more nearly to equality with each other in fucceffion, that the law fhall vary in fuch a manner, as abfolute ifochronifm requires. But in the fame manner as the fundamental property of fprings, namely that as the tenfion is, fo is the force, is determined by experiment, fo muft this property likewife be afcertained in the fame manner. Accordingly Berthoud tells us, that having attached to a balance a fpiral of very large folds, making but three turns, and whofe diameter was 15 lines, the angles through which it was wound being fucceffively 5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 60°, 120°, the counterpoifing weights in grains were 10½, 21, 32, 42, 54, 65, 76, 88, 99, 134, 278. The fame fpring forming very fmall fpires, making five turns in 8 lines diameter, the angles through which it was wound being the fame as before, the counterpoifing weights were 11, 22, 33, 45, 56, 67, 78, 89, 100, 133, 250 grains. Thefe experiments, he tells us, were made with great care; and

and they ſhew that the ſame ſpiral, its length continuing unchanged, when folded in large and ſmall ſpires, has a ſufficient difference in its progreſſion to vary its iſochroniſm: when folded in large ſpires, according to the firſt experiment, the vibrations in larger arcs are accelerated; and by the ſecond experiment, when folded in narrow ſpires, they are rendered ſlower.

24. A ſpiral ſpring may be rendered iſochronal by a proper adjuſtment of its ſtrength and thickneſs in different parts.

25. A ſpiral ſpring which is not iſochronal, may be rendered ſuch by the addition of two auxiliary ſprings, whoſe points of quieſcence are properly adjuſted.

This was the ingenious invention of Mr. Mudge; the theory of which conſtruction is delivered in the Phil. Tranſ. for the year 1794, by Mr. Atwood, with his uſual accuracy and perſpicuity.

26. The influence of the maintaining power on the balance, in reſtoring the motion which it loſes by friction or otherwiſe, may be either conſtant or interrupted.

This depends on the eſcapement; when the action of the maintaining power is conſtant, the eſcapement is called either the Recoil or the Dead-beat; when it is interrupted, the eſcapement is ſaid to be Detached.

27. By eſcapement is underſtood the means by which the action of the wheels is applied to maintain

tain the vibration of the balance; and it confifts of the balance wheel and pallets.

28. Pallets are fmall plates or levers attached to the axis or virge of the balance, which receive the impulfe of the balance wheel produced by the maintaining power, and thus continually renew the motion which the balance lofes by friction, or other refiftance.

In a Recoil efcapement, when one tooth of the balance wheel drops off the firft pallet, the other acting tooth falls on the inclined plane of the other pallet, which meeting it obliquely caufes the balance wheel to recoil, from which circumftance this efcapement derives its name.

In the Dead-beat efcapement, when one tooth of the balance wheel drops off the inclined plane of the firft pallet, the other acting tooth immediately falls upon the convex furface of the other pallet, which furface being concentrical with the axis of the balance, the wheel continues at reft until, by the motion of the pallet or cylinder, the inclined plane of the tooth comes to act upon the face of this latter pallet or edge of the cylinder, which then, by its preffure on that edge, throws the cylinder round, and thus gives motion to the balance; then inftantly entering the cavity of the cylinder, it falls upon the concave furface, and for the fame reafon as before continues at reft, until the balance fpring drives the cylinder round in a contrary direction to what it did before, fo as that the inclined plane of the tooth may act on the fecond edge of the cylinder; which preffure throws the cylinder round in the contrary direction, and the tooth gets

gets out of the cavity, and at that inſtant the ſubſequent tooth falls upon the convex ſurface, and ſo on. From the quieſcence of the balance wheel during the interval of time that elapſes between the falling of the acting tooth on the ſurface and its preſſure on the edge of the cylinder, this eſcapement is called the Dead-beat.

In the Detached eſcapement the motion of the maintaining power is ſuſpended during almoſt the whole time of vibration; juſt at the end of the return of the balance it unlocks the wheel-work, and a tooth of the balance wheel, immediately acting on the pallet, reſtores the motion which the balance had loſt; and having given its impulſe, the wheel-work is inſtantly locked again, and the balance performs its vibration freely and diſengaged from all other parts of the machine.

29. In the eſcapement of recoil, the vibrations are quicker than if the balance or pendulum vibrated freely.

For the recoil ſhortens the aſcending part of the vibration by contracting the extent of the arc; and the reaction of the wheel accelerates the deſcending part of the vibration.

30. In the dead-beat eſcapement, the vibrations are ſlower, than when they are performed in a detached ſtate.

For the preſſure of the tooth on the ſurface of the cylinder, retards that part of the vibration which is performed while the cylinder, by the motion of the balance ſpring, revolves ſo far as to bring the tooth to the edge of the

the cylinder: and if the maintaining power be increafed, the preffure of the tooth on the cylinder may become fo great, as entirely to ftop the motion. When the tooth has communicated its impulfe to the edge of the cylinder, it moves almoft freely; and as the tooth does not yet prefs with its entire force on the next furface, the cylinder will indeed defcribe a larger arc, and therefore on that account the time may be fhortened; but when it has confumed all the impulfe of the wheel, it returns by the fole force of elafticity; now the preffure of the tooth caufes a friction which diminifhes the tendency to return to the point of reft, fo that the balance performs its vibrations flower.

31. In the efcapement of recoil, if the maintaining power be increafed, the vibrations will be performed in larger arches, but in lefs time.

Becaufe the greater preffure of the crown wheel on the pallet will caufe the balance to vibrate through larger arches; and the time, on this account will be lefs increafed, than it will be diminifhed by the acceleration of the balance by that preffure, and the diminution of the time of recoil.

32. In the efcapement of the cylinder or dead-beat, an increafe of the maintaining power renders the vibrations larger, and at the fame time flower.

Becaufe the greater preffure of the tooth on the edge of the cylinder throws it round through a greater arch; and its increafed preffure on both furfaces of the cylinder retards its motion.

33. The

33. The escapement can render those vibrations only isochronal, whose inequality proceeds from the maintaining power, and not such as are produced by external agitations.

34. The effect of external agitations on the balance may be counteracted by the double escapement.

In this escapement two equal balances are so connected, that they vibrate through equal angles, but in contrary directions; by which means, the one must always be accelerated as much as the other is retarded by any external agitation. But as Mr. Cummins observes, when balances are connected by means of teeth, there arises a resistance which, however small, when applied in this most delicate part, will tend to diminish the momentum of the balances.

35. That escapement is best in which the duration of the action of the balance wheel on the paletts is least with respect to the time of vibration.

Hence the detached escapement is the best, which appears to have been the invention of the ingenious artist Mr. Thomas Mudge, who made a watch on this construction for the late king of Spain, Ferdinand the 6th, in the year 1755.

36. The time of the vibration of the balance is increased by heat, and diminished by cold.

First, because the length of the spiral spring is increased by heat, and therefore its force diminished; and the con-

trary by cold. 2dly. The diameter of the balance is increased by heat, and therefore alſo the time of vibration; and the contrary by cold.

37. That balance is the moſt perfect which, without the compenſation of a thermometer, is moſt ſubject to the influence of heat and cold.

Becauſe the obſtructions from oil and friction act as a compenſation to the expanſion or contraction of the ſpring and balance; therefore that balance which is moſt affected, is freeſt from the influence of oil and friction.

38. The errors in the going of a watch, ariſing from the change of temperature, may be corrected by varying the length of the balance ſpring.

Nevertheleſs, as it is extremely difficult to form an iſochronal ſpiral, any variation in its length is dangerous, becauſe we ſhall thus probably loſe that point which determines its iſochroniſm.

39. The errors in the going of a watch, occaſioned by the variation of temperature, may be corrected by varying the diameter of the balance.

Fig. 21. This may be effected by dividing the rim of the balance into two or more ſeparate parts, *IK*, *LM*, *RS*, each of which is compoſed of two plates of metal of different expanſibility, rivetted together, the leaſt expanſible being neareſt the centre *H*, and carrying at one end *K*, *M*, *S*, a weight; whilſt the other is connected either with the rim of the balance, or one of its radii. Now if the temperature increaſe, the exterior plate expanding more than the interior, the compound will become more concave towards the

the centre; and consequently the end which carries the weight will approach the centre of the balance, and on that account the vibrations will be rendered quicker. At the root of each thermometer there is a screw, *I*, *L*, *R*, by which the diameter of the balance may be increased or diminished, so as to alter the time kept by the chronometer, without interfering with the adjustment for heat and cold: and if the magnitude and position of the weights be properly regulated, they will correct the error arising from the variation of the diameter of the balance, caused by the variation of temperature.

LECTURE XXIII.

1. IF the force of gravity were conſtant, and acted in parallel lines, and there were no reſiſtance from the air, a body thrown in any oblique direction would deſcribe a parabola.

2. The velocity in any point of a parabola is equal to that which would be acquired by a body falling down one fourth of the parameter belonging to that point.

3. If the plane from which the body is projected, and on which it falls, be the horizon, the axis of the parabola will be perpendicular to the horizon; the velocities equal at equal diſtances from the principal vertex; and the time of arriving at the vertex of the parabola, or at the greateſt altitude, equal to half the time of the flight.

4. If

4. If v be the velocity of projection, $g = 16\frac{1}{12}$ feet, $s =$ the fine, $r =$ the verfed fine of double the angle of elevation, $e =$ the fine of the angle of elevation to radius unity, $a =$ the amplitude on an horizontal plane, $h =$ the altitude, $t =$ the time of flight; then will $a = \frac{sv^2}{2g}$; $h = \frac{rv^2}{8g}$; and $t = \frac{ev}{g}$.

Hence the amplitude is greateft at the angle 45°, and $= \frac{v^2}{2g}$; alfo the amplitudes are equal at angles equally above and below 45°.

From the fmall diftances to which we can project bodies upon the earth's furface, the variation in magnitude of the force of gravity or its direction, will not fenfibly affect the parabolic tract of a projectile; but the deviation caufed by the refiftance of the air is fo great, that no practical conclufions can be drawn from this theory.

Mr. Robins obferves that a 24lb. ball, impelled with its ufual charge of powder, meets with an oppofition from the air equivalent to 400lb. which retards the motion fo much, that the range at an elevation of 45°. would not be above one-fifth of that given by theory. From this great refiftance, the amplitudes are not proportional to the fines of the double elevation, as in vacuo; but proportionably greater for lefs than for greater angles. Neither are the amplitudes equal, which are obtained in elevations equally above and below 45°; thofe which are correfpondent to the fmaller angles being greater; the angle alfo of greateft amplitude is not 45°, but much lefs, as 43°, 40°, 38°, 30°, or even lefs

in

in very great velocities; neither is the vertex of the trajectory in the middle, but more remote from the point of projection than the other extremity; the time alfo of defcent is longer than the time of afcent; and the part of the curve through which the body defcends is lefs curved than that through which it afcends.

5. If a ball be perfectly fpherical, a rotatory motion round any axis, during its flight through the air, will not caufe the ball to deviate from a plane perpendicular to the horizon.

Since the force of gravity acts always in a direction perpendicular to the horizon, a body projected in any direction would, if unrefifted, be always in the fame vertical circle; but Mr. Robins firft difcovered that projectiles are not only depreffed beneath the line of projection, but deflected to the right or left of that direction by fome other force. This, Mr. Robins fuppofed, was caufed by the rotation of the ball round its axis; which rotation combined with its progreffive motion, would caufe the furface of the bullet to ftrike againft the air very differently from what it would do, were there no fuch rotation; but Mr. Euler has demonftrated in the following manner, that thefe two changes fo counteract each other's effects, that the action of the air upon any point of the ball, fuppofing it perfectly fpherical, is the fame as if the ball had no fuch rotatory motion:

Fig. 22. Let a plane, reprefented here by the paper, touch the ball in the point *A*, and let the air impinge on this point *A* in the direction *PA*. Let *PA* exprefs the

velocity

velocity of the air, and from the point P let fall the perpendicular PQ upon the tangent plane, and draw AQ.

If the ball did not turn round, the force of the stroke of the air would be as the square of the line PA, expressing its velocity, into the square of the sine of the angle PAQ, which the direction of the air's motion PA makes with the surface of the ball in the point A. Since the sine of the angle PAQ is expressed by $\frac{PQ}{PA}$, the force of the air on the point A will be as $PA^2 \times \frac{PQ^2}{PA^2}$, that is, as PQ^2. But if the ball turn round, its centre remaining unmoved, the point A can have no other motion than that whose direction lies in the touching plane. Let Aa be the direction of the motion of the point A, and let Aa express its velocity in that direction, with which it impells the air; the effect on the point A is the same, as if the point A were at rest, and the air besides its motion in the direction PA, had also a motion whose direction and magnitude were denoted by the line Pp, equal and parallel to Aa; but these two motions are equivalent to one motion whose direction and magnitude are expressed by Ap, the diagonal of the parallelogram, whose sides are PA and Pp; therefore the action of the air on the point A will be the same, as if the point stood still, and the air rushed against it with the velocity pA; from p let fall the perpendicular pq on the tangent plane, and the force of the air

air, in this case, will be equal to $pA^2 \times \frac{pq^2}{pA^2} = PQ^2$, the same quantity as before.

6. If a ball be perfectly spherical, the centre of gravity's not coinciding with the centre of magnitude will not cause any deviation in its course.

From the two last articles it appears, that the deviation of a bullet from the vertical plane is produced by its not being perfectly spherical. For if the bullet be irregular in its figure, and have a rotatory motion, and the axis of rotation does not coincide with the line of flight, the resistance, which the irregularity of the surface produces, will cause the ball to deviate from the course in which it would otherwise have continued. For since the air is condensed before the ball, any part projecting from the spherical surface of the ball, will meet with a greater resistance in the denser medium before the ball, than in the rarer medium behind it; therefore the tendency that it has to recede in the direction of the tangent, by the reaction of the denser part of the medium, will not be corrected by the contrary reaction behind the ball; it will therefore, by this reaction compounded with the direct motion of the ball, describe an intermediate line, deviating more or less from the direct course.

The only case in which the rotatory motion produces no error, is when the axis of rotation coincides with the line of the bullet's flight; because though by the casual irregularity of the foremost surface of the ball, or any other accident, the resistance should be greater on one side than on another;

another; yet as the place where this greater reſiſtance acts, muſt perpetually ſhift its poſition round the line in which the bullet flies, the direction which this inequality would occaſion, if it acted conſtantly with the ſame tendency, is now continually rectified. Hence the uſe of rifled barrels, whoſe effect is to render the axis of rotation of the bullet coincident with the line of flight.

LECTURE XXIV.

1. WHEN a body in free ſpace deſcribes a curve returning into itſelf, the centripetal force is equal to the centrifugal force; and they are called by one common name, Central Forces.

Every body that deſcribes a curve is acted on by two forces, a projectile and centripetal force; the projectile force is that, with which the body would run out in a tangent to its orbit, if there were no centripetal force to prevent it; and the centripetal force is that which, conſtantly acting on the body, turns it from its rectilineal courſe, and bends it into a curve.

The centrifugal force is the force by which the body would be carried off from the centre, were it not prevented by the centripetal force; and therefore the centrifugal and centripetal forces are always expreſſed by the ſame line.

2. When

2. When a body revolves on an axis, the centrifugal force of any of the particles may be to the centripetal in various affigned proportions of inequality.

For though the centrifugal force is in this cafe meafured by the nafcent fubtenfe of the angle of contact, the centripetal force is not.

3. The centripetal and projectile forces are heterogeneous, and cannot be compared.

For the action of the one is inceffant, and of the other impulfive.

Fig. 23. Let a body move in the circle *Abd*, the centripetal force is to the projectile as *Bb*, the fubtenfe of the evanefcent angle of contact, to the tangent *AB*, or as *Aa* to *ab*, that is, as the evanefcent chord *Ab* to *bd*; or ultimately, as the nafcent chord *Ab* to the diameter. Therefore the projectile force is indefinitely greater than the centripetal. In fact thefe forces differ in the fame manner as preffure and momentum. Hence therefore it feems improper to fay, that the centrifugal force is part of the projectile; whereas, in truth, it is no other than the meafure of the force, by which the body continually endeavours to recede from a point not in the line of projection.

4. Bodies which are retained in curves by a force tending to a given centre, defcribe areas proportional to the times round that centre.

5. Bodies agitated by a centripetal force defcribe curves concave towards the centre of force.

It does not neceſſarily follow, that the centripetal force ſhould cauſe the body always to approach the centre; it may continue to recede from the centre of force, notwithſtanding its being drawn by that force; but this property muſt always belong to its motion, that the line which it deſcribes be concave towards the centre to which the force is directed.

6. The plane of the curve paſſes through the centre of force.

7. A body which moves in a curve and deſcribes areas proportional to the times round any point, is actuated by a central force tending to or from that point.

8. Central forces are as the verſed ſines of the arcs deſcribed in equal times, which converge to the centre of force, and biſect the chords, when thoſe arcs are diminiſhed indefinitely.

9. The projectile velocity in any point of a curve is reciprocally as the perpendicular let fall from the centre of force on the tangent to that point of the curve.

10. The angular velocity at the centre of force is reciprocally as the ſquare of the body's diſtance from that centre.

11. In a circle the velocity is uniform, if the centre of force coincide with the centre of the circle.

12. If bodies deſcribe the peripheries of circles with a uniform motion, the centripetal forces tend to

to the centres of the circles, and are to each other as the ſquares of the arcs deſcribed in equal times, applied to the radii of the circles.

13. The central forces are alſo as the ſquares of the velocities directly, and the radii of the circles inverſely.

14. The central forces are alſo in a ratio compounded of the ratio of the radii directly, and the duplicate ratio of the periodical times inverſely.

Hence we may compare the central force of a body revolving in the periphery of a given circle with the force of gravity, by finding the periodical time of a body projected ſo that its centripetal force near the ſurface of the earth may be equal to the force of gravity; for as the radius of the given circle, divided by the ſquare of the periodical time in that circle, is to the radius of the earth divided by the ſquare of the periodical time found, ſo is the central force in the given circle to the force of gravity: let r be the radius of the given circle, t the periodical time in that circle; d the diameter of the earth, $p = 3.1416$ the periphery of a circle whoſe diameter is unity, g the ſpace deſcribed by a falling body in one ſecond; then is g likewiſe the ſubtenſe of an arc deſcribed in a ſecond of time, which arc is therefore $= \sqrt{dg}$; therefore $\sqrt{dg} : dp$, the circumference of the earth :: $1''$: $\frac{dp}{\sqrt{dg}}$, which is the periodical time of a body revolving at the earth's ſurface, with

with a centripetal force equal to the force of gravity; therefore the central force is to the force of gravity: : $\frac{2rp^2}{gt^2}$: 1 : : $\frac{1,2272r}{t^2}$: 1. Thus if a ball of one ounce tied to a ſtring, be whirled about in a circle whoſe radius is 2 feet, and it deſcribe the periphery in ½ a ſecond, the centrifugal force is to the force of gravity, as 9.817 to 1; and therefore the force, with which the ſtring is ſtretched, is 10 ounces nearly.

15. The central forces are as the diſtances and quantities of matter directly, and inverſely as the ſquares of the periodic times.

For, the quantity of matter being different, and every thing elſe given, the central force is to the force of gravity in a given ratio; but the force of gravity is as the quantity of matter; and if the quantity of matter be the ſame, and the other quantities varied, the central forces are as the radii directly, and the ſquares of the times inverſely.

16. If the ſquares of the periodic times be as the cubes of the diſtances, the central forces will be as the quantities of matter directly, and inverſely as the ſquares of the diſtances.

17. If two bodies revolve round their common centre of gravity, they will ſuſtain each other, and continue at the ſame diſtance from each other, however great their velocity may be. But if they revolve round any other point, that body whoſe

centrifugal

centrifugal force is greater, will carry off the other with it.

18. If a body projected in a given direction be constantly drawn towards two fixed points, which are not both in the same plane with the direction, it will describe equal solids in equal times about the right line joining the said points.

Fig. 24. Let the time be divided into any number of equal parts, and in the first moment let it describe the line AQ; in the second moment, if not prevented, it would proceed to describe the line QT equal to AQ, but at Q it is acted on by centripetal forces tending to the centres C and B; let these forces be expressed by the lines QS, QV, which would be described by them, while the body would be carried from Q to T; complete the parallelepiped whose sides are the lines QS, QV, QT; the body by the joint action of these forces, will describe QD, the diagonal of the parallelepiped. But the solid $QCBA = QCBT$, because they stand on the same base QCB, and have the same altitude (for the right line TA cuts the plane QBA in Q, and $TQ = QA$); also the solid $QCBT = QCBD$, because they stand on the same base, and are between the same parallel planes QCB, DT. In the same manner it is manifest, that equal solids will be described, in other equal moments of time, round the same points. Therefore if the number of parts of time, and of the right lines AQ, QD, be indefinitely increased, the path of the body will ultimately become a curve, and the body impelled by continued forces will describe round the points C, B, solids proportional to the times.

Conversely

Converſely, if theſe ſolids *QCBA*, *QCBD* deſcribed in equal times be equal, the line *TD* will be parallel to the plane *QCB*; and therefore the body will be urged by forces *QS*, *QV* tending to the points *C*, *B*.

The law here laid down explains the motion of the moon and ſatellites, which are attracted towards two different centres.

19. When different bodies are carried round in a vortex, the matter of which is denſer than the bodies, they will continually approach the centre of force in a ſpiral line; if it be not ſo denſe, they will recede from the centre in a ſpiral; if it be of the ſame denſity, they will deſcribe concentric circles, and their periodic times will be equal.

20. If a rigid ring endeavour to recede from a plane paſſing through the centre of the ring, by revolving on its diameter, which is the common interſection of the circle and the plane, the ſum of the centrifugal forces with which all the particles in the ring endeavour to move the plane of the circle about its diameter, will be to the ſum of the forces of all the particles collected in the point which is fartheſt from the plane, as 1 to 2.

See Newton's Princip. Lib. 3. Lemma 1.

21. If a rigid ring *nqNQ* revolve with two motions, one in its own plane, and the other about the diameter *QTq*; and if a motive force acting at the

the point D be ſuppoſed equivalent to the whole motive force acting upon the ring; then half this force is efficacious in accelerating the motion of the point Q, in a direction perpendicular to the plane of the ring, and the other half is conſumed in counteracting the centrifugal force, ariſing from the motion of the particles of the ring about a momentary axis PTp.

Fig. 25. For in the great circle nh, let a point h be taken indefinitely near to n, and in the ring a point r ſo that nh and Qr may repreſent the angular velocities about the diameter and centre of the ring.

Let d and c repreſent thoſe velocities, and r the radius of the ring. Draw rs perpendicular to the plane of the ring, and meeting the great circle hQs in s; then will rs repreſent the accelerating force of the point Q, perpendicular to the plane of the ring; but $rs : nh :: Qr :$ rad. (r) therefore $rs = \frac{cd}{r}$; and if R be $=$ the matter in the ring, the whole efficacious motive force $= \frac{cd}{r} \times \frac{1}{2} R$.

The momentary axis PTp is in a plane perpendicular to the plane of the ring, and paſſing through Qq; make PT $=$ radius of the ring, and draw Pr perpendicular to Qq. The velocity of the point P in a circle parallel to nh, in conſequence of the motion about the diameter, is to the velocity of n in the circle nh, as the radii of thoſe circles, or as Pr to radius, and therefore is $= d \times Pr$ divided by radius. In like manner, the velocity of P, in conſe-

 quence

quence of the motion round the centre, is to the velocity of Q in the plane of the ring, as Tr to radius, and therefore is $= c\times Tr$ divided by radius; but since P is the momentary pole, and therefore quiescent, these two motions are equal; and therefore $c : d :: Pr : Tr$: or $Pr = \frac{cr}{\sqrt{c^2+d^2}}$, and $Tr = \frac{dr}{\sqrt{c^2+d^2}}$. In fig. 26. let PT represent the momentary axis, and QEN a quadrant of the ring; from any point E of the ring draw Ev perpendicular to PT, and vw perpendicular to QT. The centrifugal force of E : centrifugal force of N : : Ev : NT, or the centrifugal force of $E =$ centrifugal force of $N \times \frac{Ev}{NT} = \frac{c^2+d^2}{r} \times$ particle $E \times \frac{Ev}{NT}$, because the velocity of $N = \sqrt{c^2+d^2}$. But the efficacious part of this force, in a direction perpendicular to the plane of the ring $=$ whole $\times \frac{vw}{Ev}$; and a force acting at Q equivalent to this $=$ the whole $\times \frac{vw}{Ev} \times \frac{Tx}{TQ} = \frac{c^2+d^2}{r} \times E \times \frac{Ev}{NT} \times \frac{vw}{Ev} \times \frac{Tx}{TQ} = \frac{c^2+d^2}{r} \times E \times \frac{vT\times Pr\times Tx}{TQ^3}$. Now if great circles be conceived drawn through P, Q, and P, E, (by Sph. Trig.) cos. PE (vT) $\times$ Rad. (TQ) $=$ cos. PQ (Tr) $\times$ cos. QE (Tx). Therefore a motive force at Q, equivalent to the motive

motive efficient centrifugal force of $E = \frac{c^2+d^2}{r} \times E \times \frac{Tr \times Pr \times Tx^2}{TQ^4} = \frac{c^2+d^2}{r} \times E \times \frac{dc}{c^2+d^2} \times \frac{Tx^2}{TQ^2} = \frac{dc}{r} \times E \times \frac{Tx^2}{TQ}$; therefore the ſum of all theſe quantities = the motive force at Q equivalent to the ſum of all the efficient centrifugal forces, or the centrifugal force of the ring. But the ſum of all theſe quantities $= \frac{dc}{r} \times \frac{1}{2} R$, becauſe the ſum of all the ſquares of the ſines Tx in a circle is to the ſum of the ſquares of as many ſemi-diameters TQ, as 1 to 2.

Hence the motive force at Q equivalent to the ſum of all the efficacious centrifugal forces, is expreſſed by the ſame quantity as the force at Q equivalent to the whole motive efficacious force on the ring. This elegant demonſtration is Mr. Brinkley's. See Vol. 7. Tranſ. R. Iriſh Acad.

On this propoſition depends Mr. Landen's ingenious detection of Newton's error in calculating the preceſſion of the Equinoxes.

LECTURE XXV.

1. To produce the motion of a machine, the power muſt exceed the weight; under which is compriſed friction, the rigidity of ropes, and the inertia of the parts of the machine.

2. If a cylinder whoſe weight is w revolve round its axis by the action of a force p applied to the circumference, the ſpace deſcribed in t ſeconds will be $= gt^2 \times \frac{2p}{2p+w}$.

Fig. 27. Let DEB be the cylinder, revolving on its axis S, by the action of p on the point D, and let A be the centre of gyration; the inertia of the matter of the cylinder whoſe weight is w, is equivalent to the maſs w, collected in the point A; and this will be equivalent to the weight $w \times \frac{SA^2}{SD^2}$ collected in the point D (Art. 22. Lect. 19.) $= \frac{1}{2} w$,

becauſe

becaufe $SA = \frac{SD}{\sqrt{2}}$. But the accelerating force is equal to the moving force p divided by the weight, that is $= \frac{p}{\frac{1}{2}w}$.

If p be a weight poffeffing inertia, this inertia muft be added to the mafs moved, and the accelerating force will then be

$$\frac{p}{p+\frac{1}{2}w} = \frac{2p}{2p+w}, \text{ and } S = gt^2 \times \frac{2p}{2p+w}.$$

3. If two bodies p and w hang over a fixed pulley, of which p is the greater, then will the fpace defcribed $S = \frac{p-w}{p+w} \times gt^2$, the inertia of the pulley not being confidered; and the velocity of the power and weight $= 2gt \times \frac{p-w}{p+w}$.

For $p-w$ is the moving force $= M$, and $p+w$ the quantity of matter moved $= Q$; but $S = \frac{M}{Q} \times gt^2$; and $V = 2gtF$.

If the pulley be a cylinder whofe weight is Q, its inertia will be the fame, as if $\frac{1}{2}Q$ were uniformly accumulated in its circumference; therefore the accelerating force will be

$$\frac{p-w}{p+w+\frac{1}{2}Q} = \frac{2p-2w}{2p+2w+Q} = F, \text{ and } S = gt^2F.$$

4. If a weight w is to be raifed by a power p, by means of a fixed and moveable pulley, then will

$S=$

$S = \frac{4p-2w}{4p+w} \times gt^2$, the inertia of the parts of the machine not being confidered.

Since the weight w acts againft p with a force $= \frac{1}{2}w$, the whole moving force $= p - \frac{1}{2}w$; and fince the weight w moves with a velocity only one-half of that with which p moves, its inertia referred to p's motion will be $\frac{1}{4}w$; therefore the inertia of the whole mafs $= p + w$, and $S = \frac{p-\frac{1}{2}w}{p+\frac{1}{4}w} \times gt^2$.

If the pullies be cylinders, and the weight of each $= Q$, the inertia of the fixed pulley will be $= \frac{1}{2} Q$, by Art. 2; the inertia of the moveable pulley will alfo be the fame, as if $\frac{1}{2} Q$ were accumulated in its circumference; and fince the velocity of the circumference is to that of p as 2 : 1, its inertia will be equivalent to a mafs $\frac{Q}{2 \times 4}$ moving with the velocity of p. See Art. 22. Lect. 19. Hence the whole mafs moved will be $= p + \frac{1}{4}w + \frac{1}{2}Q + \frac{1}{8}Q = \frac{8p+2w+5Q}{8}$, and the accelerating force $= \frac{8p-4w}{8p+5Q+2w} = F$, and the fpace defcribed by p from reft in t feconds $= gt^2 \times F$.

5. If a body is to be raifed a given height by means of an inclined plane, by a given power, the time of its afcent will be the leaft poffible, when the length : height : : twice the weight to the power.

Fig. 28.

Fig. 28. Let AB the length of the plane $= x$, the given height $= a$; the force of descent along the plane is $= \frac{a}{x} w$, therefore the moving force $= p - \frac{a}{x} w$; and the accelerating force $= \frac{px - wa}{x \times \overline{p+w}}$. Now the time $= \sqrt{\frac{S}{gF}} =$

$\frac{x \sqrt{p+w}}{g^{\frac{1}{2}} \sqrt{px - wa}}$ a minimum, or $\frac{x^2}{px - wa}$ a minimum. Therefore

$\frac{2x\dot{x} \times \overline{px - wa} - p\dot{x}x^2}{\overline{px - wa}|^2} = o$; but when a fraction vanishes, its numerator $= o$; hence $2px^2 - 2awx - px^2 = o$, or $px^2 = 2awx$; therefore $x : a :: 2w : p$.

See a very elegant Geometrical demonstration of this proposition in Maclaurin's View of Newton's Discoveries, B. 2. ch. 3.

6. If a, b be the distances of the power and weight from the fulcrum C of the lever pw, then will the initial velocity of the power $= 2gt \times \frac{\overline{pa - wb}}{pa^2 + wb^2} \times a$, the weight and inertia of the lever itself not being considered.

Fig. 29. If the power and weight were equal, p would be equal to $\frac{bw}{a}$; the power therefore which moves the lever will be $= p - \frac{bw}{a}$. Since this power applied to the point p acce-

lerates

lerates the maſſes p and w, the maſs to be ſubſtituted for w in the point p, muſt be $= \frac{b^2 w}{a^2}$, in order that this maſs, at the diſtance a, may be equally accelerated with the maſs w at the diſtance b. The power therefore $p - \frac{bw}{a}$, will accelerate the quantities of matter $p + \frac{b^2 w}{a^2}$, and the accelerating force $= \frac{p - \frac{bw}{a}}{p + \frac{b^2 w}{a^2}} = \frac{pa^2 - wba}{pa^2 + wb^2}$. But the velocity $V = 2gtF = 2gt \times \frac{pa^2 - wab}{pa^2 + wb^2}$.

If $a : b :: n : 1$, the force which accelerates $p = \frac{pn^2 - wn}{pn^2 + w}$.

If the inertia of the moving force be alſo $= o$, as in muſcular action, the force accelerating p will be $= \frac{pn^2 - wn}{w}$.

If the maſs moved has no weight, but poſſeſſes inertia only, as when a body is moved along an horizontal plane, the force which accelerates $p = \frac{pn^2}{pn^2 + w}$.

7. The velocity of the weight will be $= 2gt \times \frac{pab - wb^2}{pa^2 + wb^2}$.

8. The ſpace deſcribed by the power, in the time t, will be $= gt^2 \times \frac{pa^2 - wab}{pa^2 + wb^2}$.

9. If

9. If a and b denote the radii to which the power and weight are applied in an axle in the wheel, the momentum communicated to the weight will be a maximum, when the weight $= p \times \sqrt{\frac{a^4}{b^4} + \frac{a^3}{b^3} - \frac{a^2}{b^2}}$, the inertia of the machine not being confidered.

For the velocity of the weight $= 2gt \times \frac{pab - wb^2}{pa^2 + wb^2}$; and the moment generated in w, in the time t, will be expreffed by $2gt \times \frac{pwab - w^2 b^2}{pa^2 + wb^2}$; and as this is to be a maximum, we have $ba^3 p^2 - 2b^2 a^2 pw - b^4 w^2 = o$.

If $a : b :: n : 1$, $w = p \times \sqrt{n^4 + n^3 - n^2}$. And if the radius of the axle be equal to that of the wheel, that is, if $n = 1$, the weight will be $= p \times \sqrt{2 - 1}$, and therefore will be about $\frac{5}{12}$ parts of the moving force. See Atwood's Rect. Motion, Page 249.

10. In any machine, the weight will be moved with the greateft velocity, when the velocity of the power is to that of the weight as $1 + p\sqrt{\frac{p+w}{w}} : 1$, the inertia of the machine not being confidered.

For the machine may be confidered as reduced to a lever whofe arms are a and b, and the velocity of the weight $= 2gt \times \frac{pab - wb^2}{pa^2 + wb^2}$, a maximum, where b is variable; put its fluxion $= o$, and we have $a = \frac{bw + \sqrt{b^2 w^2 + b^2 pw}}{p}$.

If $p = w$, that is, if the weight moved be equal to the power, then $a : b :: 1 + \sqrt{2} : 1$.

11. If motion be communicated to a heavy cylinder whofe weight is w and radius r, revolving on its axis in a vertical plane, by a fmall weight p acting at the diftance d from the axis, and the velocity of d be v; the momentum of p detached, and moving with any other velocity V, will be to its momentum when attached to the point d, as $pV : v \times \frac{wr^2}{2d^2}$.

Suppofe $p = 1$ oz. or nearly the weight of a mufket ball, $V = 1700$ feet, or the greateft velocity with which a mufket ball can move, $r = 10$ feet, and $d = 1$ foot, then will $w = 17\,p$. That is, if the wheel be feventeen times the weight of the ball, the impact of the ball when attached to the point d of the wheel, and moving with the velocity of one foot in a fecond, will be equal to the utmoft force of a mufket ball.

But fince mufket balls produce their effect not merely by inftantaneous impact, but by penetrating into and through fubftances, and therefore the time of their action is to be taken into the account, the forces will be as the fquares of the velocities, or as $pV^2 : v^2 \times \frac{wr^2}{2d^2}$; and therefore when they are equal, in the above cafe, $w = \frac{2p \times 1700^2}{100} = 57800\,p = 3612$ pounds. See Atwood on Rect. Motion pag. 264.

12. The

12. The time in which a given weight f, acting at the circumference of the cylinder whose radius is r, in order to generate the velocity v in the point d, will be $= \frac{rv}{d} \times \frac{f + \frac{1}{2}w}{32\frac{1}{6}f}$.

For the quantity of matter moved $= \frac{w \times \overline{r\sqrt{\frac{1}{2}}}^2}{r^2} = \frac{1}{2}w$; the accelerating force $= \frac{f}{f+\frac{1}{2}w}$; but when the velocity of d is v, the velocity of r will be $\frac{rv}{d}$; and the time in which the above accelerating force will generate the velocity $\frac{rv}{d}$, will be $\frac{rv}{d} \times \frac{f+\frac{1}{2}w}{32\frac{1}{6}f}$. Let $d = 1$ foot, $r = 10$ feet, $f =$ 20 pound, $v = 1$ foot in a second, and $w = 3612$, then will the time $= \frac{10}{32\frac{1}{6}} \times \frac{20+1806}{20} = 28''\frac{1}{2}$ nearly. So that if a man's arm acting at the circumference of the cylinder, could exert a force equal to 20 pound, it would in $28''\frac{1}{2}$ generate such a momentum in a musket ball, attached to the radius, at the distance of a foot from the axis, as would be equal to the force of a musket ball discharged with its utmost velocity. See Atwood ubi supra.

13. When an animal moves a weight, its force is as the square of the difference between the velocity with which it moves, and the utmost velocity with which it is capable of moving, when not impeded by any weight.

For if the velocity of the animal were equal to its utmost velocity, it would have no force whatever to move the

weight; it is therefore the difference between this velocity and that with which it actually moves, which is efficient; but the pressure of the animal against the weight is supposed to be uniform and constant; its action therefore is analogous to the constant pressure of a fluid, which is as the square of the efficient velocity.

It is admitted, that the pressure of the animal is not, in fact, actually uniform during the whole time in which it acts, but it is nearly so, and we must make the hypothesis of its uniformity, in order to enable us to approximate to the true nature of its action.

14. If a be the absolute force of an animal, c the utmost velocity with which it can move, when not impeded, v the velocity with which it moves, when impeded by the weight, then will its force p to move the weight be $= a \times \overline{1 - \frac{v}{c}}\Big|^2$.

For $a : p :: \overline{c - o}|^2 : \overline{c - v}|^2$, therefore $p = a \times \overline{\frac{c - v}{c}}\Big|^2$.

15. The utmost velocity with which an animal, not impeded, can move, is to the velocity with which it moves, when impeded by a given weight, as the square root of its absolute force to the difference of the square roots of its absolute and efficient forces.

For $1 - \frac{v}{c} = \frac{\sqrt{p}}{\sqrt{a}}$; therefore $c = \frac{v\sqrt{a}}{\sqrt{a} - \sqrt{p}}$.

16. The force of an animal is greatest, when the velocity with which it moves is one-third of the

the greateſt velocity with which it is capable of moving, when not impeded.

For the moment of the animal $= av \times \overline{1 - \frac{v}{c}}|^2$; which is a maximum, when $v = \frac{1}{3} c$.

According to Euler, the force of a man, at reſt, is = 60 pounds; and the utmoſt velocity with which he can move, is 6 feet in a ſecond. Hence the greateſt force which a man can exert, when in motion, will be equivalent to 27 pounds; and he will then move at the rate of 2 feet in a ſecond.

The ſtrength of a horſe is ſuppoſed to be ſeven times greater than that of a man; and therefore = 420 pounds, when at reſt; and his greateſt velocity, at a medium, is equal to 12 feet in a ſecond; therefore the *maximum* of his action, when in motion, will be $186\frac{2}{3}$ pounds; and he will then move at the rate of 4 feet in a ſecond.

LECTURE

LECTURE XXVI.

1. THE resistance which a body suffers from the fluid medium through which it is impelled, depends on the velocity, form, and magnitude of the body, and on the inertia and tenacity of the fluid.

Fluids resist the motion of bodies through them, 1st. By the inertia of their particles. 2dly. By their tenacity or adhesion of their particles; and 3dly. By the friction of the body against the particles of the fluid. In perfect fluids the latter causes of resistance are very inconsiderable, and therefore are not taken into account; but the former is very considerable, and it obtains equally in the most perfect as in the most imperfect fluids.

2. In fluids uniformly tenacious, the resistance is as the velocity with which the body moves.

Since

Since the cohesion of the particles of the fluid is always the ſame in the ſame ſpace, whatever be the velocity, the reſiſtance, from this coheſion, will be as the ſpace deſcribed in a given time, that is as the velocity.

3. In a fluid whoſe particles move freely without diſturbing each others motions, and which flows in behind as faſt as a plane body moves forward, ſo that the preſſure on every part of the body is the ſame, as if the body were at reſt, the reſiſtance will be as the denſity of the fluid.

4. On the ſame hypotheſis, the reſiſtance will be as the ſquare of the velocity.

For the reſiſtance muſt vary as the number of particles which ſtrike the plane in a given time, multiplied into the force of each againſt the plane; but both the number and the force is as the velocity.

This proof ſuppoſes, that after the body ſtrikes a particle, the action of that particle entirely ceaſes; but the particles, after they are ſtruck, muſt neceſſarily be made to diverge, and act upon the particles behind them, which makes ſome difference between theory and experiment.

5. If a plane ſurface move perpendicularly forward, the reſiſtance will be as the area.

6. If the plane move obliquely, the reſiſtance, perpendicular to the plane, will vary as the ſquare of the ſine of the angle of inclination.

For the number of particles which ſtrike the plane, will be diminiſhed in the ratio of radius to the ſine of inclination,

tion, and the force alſo of each particle will be diminiſhed in the ſame proportion.

7. The reſiſtance of the ſame plane in the direction of its motion will vary as the cube of the ſine of the angle of inclination.

For the whole reſiſtance in a direction perpendicular to the plane, and which is expreſſed by the ſquare of the ſine of inclination, is to the reſiſtance in the direction of the plane's motion, as radius to the ſine of inclination.

8. If a rectangular plane revolve round an axis, and m and n be the diſtances of the ſides from the axis parallel to them, of which m is the leaſt, then will d the diſtance of the centre of reſiſtance from the axis be $= \sqrt[3]{\dfrac{\overline{n^2+m^2} \times \overline{n+m}}{4}}$.

The effect of the reſiſtance of any particle p of the plane to oppoſe the force which cauſes the revolution of the plane, is as the reſiſtance of that particle, multiplied into its diſtance x from the axis of motion; that is, becauſe the reſiſtance varies as the ſquare of the velocity, as $px^2 \times x = px^3$; and the reſiſtance of the whole ſurface is equal to the ſum of all the px^3. For the ſame reaſon, the reſiſtance of the whole $= d^3a$, a being the area of the plane; that is, $d^3a =$ ſum of all the $px^3 = \dfrac{n^4-m^4}{4\overline{n-m}} \times a = \dfrac{\overline{n^2+m^2} \times \overline{n+m}}{4} \times a$. See Vince's Lectures.

9. If a ſphere and cylinder of the ſame diameter move in the direction of its axis, the reſiſtance to the

the motion the cylinder will be double of that to the motion of the globe.

Fig. 30. Let *AFE* be a diameter of the end of a cylinder, parallel and equal to *BD* a diameter of the sphere *BFDG*, whose centre is *C*, and *CF*, *DE*, *BA* perpendicular to *AE*; draw *QP* parallel to *CF*, and let it represent the force with which a particle of the fluid would act perpendicularly at *v*, the end of the cylinder, in which case no part is lost. Now conceiving the same particle to act upon the globe at *P*, part of its effect will be lost by the obliquity of the stroke; draw *PR* a tangent to the sphere in the plane *PCF*, and *QR* perpendicular to it; draw also *RS* perpendicular to *QP*, and produce *QP* to *m*. Resolve the whole force *QP*, into *RP*, *QR*; the part *RP* in the direction of the tangent will be inefficient, and the part *QR* only effective; resolve this into *QS*, *SR*; then *QS* is employed in opposing the motion of the globe, and *SR* perpendicular to it will be destroyed by an equal and opposite force, on the other side. Hence the force with which the point *v*, at the end of the cylinder, is retarded, is to the force with which the corresponding point *P* on the globe is retarded, as $QP : QS$, that is, as $QP^2 : QR^2$, that is, (by sim. triangles) as PC^2 to Pm^2, or PC to $\frac{Pm^2}{PC}$, or (taking $vn = \frac{Pm^2}{PC}$) as PC to vn, or vm to vn; consequently the whole resistance on the cylinder : on the globe : : the sum of all the vm's : the sum of the vn's. Draw nr parallel to mC. Now $vm : vn :: PC^2 : Pm^2$, therefore vm or $CF : mn$ or $Cr :: PC^2$ or

 AF^2

$AF^2 : PC^2 - Pm^2$, or rn^2; the locus therefore $AnCE$ of all the points n is a parabola. Conceive now this figure to revolve about FCG, then will AE generate the end of the cylinder, and BFD that half of the globe which is resisted; also the sum of all the vm's will be the solid generated by the parallelogram $AEDB$, and the sum of all the vn's will be the solid generated by the inscribed parabola ACE, which solids are as 2 : 1 *.

When the bodies move slow, the proposition will nearly hold in air and water, but more accurately in air, because the particles move more freely, and less disturb each other's motions; but when the motion is greater, considerable aberrations will arise; both from the mutual disturbance of the particles, and the fluid not flowing in behind, as fast as the body moves forward: also in the air, a new cause of aberration will arise from the condensation of the fluid before the body. Sir I. Newton supposes, that in a continuous non-elastic fluid, infinitely compressed, the resistance of a sphere and cylinder are equal; but this appears to be an error in theory as well as in fact, for the Lemma on which he founds his inference has been justly called in question; and, when the motion is slow in water, the fluid may be supposed to be nearly of that nature which Newton supposes; yet the resistances are as nearly coincident with theory, as when the motion is in air; thus Borda found the resistance of a sphere moving in water to

* A paraboloid is half of the circumscribed cylinder. See Jacquier's Newton, Vol. 2. No. 193.

be

be to that of its great circle as 1 to 2,508; and in air the reſtances were as 1 to 2,45.

Experiment gives the ratio of the reſiſtances greater than that of 2 to 1; and the reaſon ſeems to be this: theory ſuppoſes the action of every particle of the fluid to ceaſe the inſtant it makes its impact on the ſolid; now this is not really the caſe, as we have already obſerved; and ſince the particles, after impact on the ſphere, ſlide along its curved ſurface, and ſo eſcape more readily than along the face of the cylinder, the error will be greater in the cylinder, that is, the greater reſiſtance will exceed theory more than the leſs. It is alſo to be obſerved that the difference between the reſiſtances of the globe and cylinder in water is greater than in air, directly contrary to what might be inferred from Newton's reaſoning, which ſuppoſes them equal in a continuous fluid, but in the ratio of 1 to 2 in a rare fluid.

10. The reſiſtance of a plane ſurface moving perpendicularly in a fluid, is equal to the weight of a column of the fluid, whoſe baſe is the reſiſted ſurface, and altitude equal to the ſpace through which a body falling from reſt by the force of gravity acquires the velocity of the moving plane.

The reſiſtance to the ſurface is meaſured by the number of equal particles impelling it directly in a given time, and the velocity of each conjointly; or when eſtimated in an inſtant, by one lamina of the fluid, whoſe area is equal to that of the given ſurface, multiplied into the velocity. But if this lamina were to impinge directly againſt the loweſt

lamina of the column, with a velocity equal to that with which the fluid would flow out, if an aperture were made in the bottom, it would manifeftly prevent the efflux of the fluid, or be equivalent to the weight of the column perpendicularly incumbent, which generates the velocity of the efflux.

The impact of a fluid cannot be properly faid to be equal to a weight; but confidering them as forces, whofe magnitudes are eftimated by their contemporary effects, when thefe effects counteract each other, the forces may be faid to be equal. See Parkinfon's Nat. Phil. p. 174.

11. If a cylinder move in the direction of its axis, the refiftance will be equal to the weight of a cylinder of the fluid of the fame bafe with the folid, and whofe altitude is equal to the fpace through which a body muft fall to acquire the velocity of the body: and the refiftance to a globe of the fame diameter will be half that weight.

For the anterior plane furface only of the cylinder will communicate motion to the fluid, becaufe the curved furface is parallel to the direction wherein the body moves: let d be the diameter of the cylinder, p the periphery of a circle whofe diameter is unity, z the fpace through which a body muft fall to acquire the velocity with which the cylinder moves; then will $\frac{d^2p}{4}$ be the bafe of the cylinder, and $\frac{zd^2p \times 1}{4}$ will be the weight of the fluid which mea-

fures

ſures the reſiſtance, when unity expreſſes the denſity of the fluid.

Since the reſiſtance of a ſphere is half that of a cylinder, it will be equal to $\frac{zpd^2 \times 1}{8}$.

12. The force of reſiſtance which is oppoſed to a ſphere, is to the force which would deſtroy the ſphere's whole motion, in the ſame time in which it deſcribes uniformly $\frac{8}{3}$ parts of its diameter, as the denſity of the fluid to the denſity of the ſphere.

Let the ſpecific gravity of the ſphere be to that of the fluid as $n : 1$, w its weight, and v its velocity; the time of deſcribing $\frac{8}{3} d$ uniformly, with the velocity v, is equal to the time of deſcribing half that ſpace or $\frac{4}{3} d$ from reſt, by the conſtant action of F; and (Lect. 18. Art. 13.) $F : w :: \frac{v^2}{\frac{4}{3}d} : \frac{v^2}{z}$, and $F = \frac{3wz}{4d}$. But $w = \frac{pd^2n}{6}$, and conſequently $F = \frac{pd^2zn}{8}$. But the reſiſtance of the ſphere $= \frac{pd^2z}{8}$; conſequently F : reſiſtance of the ſphere $:: n : 1$.

13. The motion of bodies in reſiſting mediums which at firſt is accelerated, will at length become uniform.

14. As a body deſcends in a fluid, it continually adds more weight to the fluid until it has acquired its greateſt velocity, at which time the weight added to the fluid is equal to the weight of the body.

15. The

15. The greateſt velocity which a globe can acquire by deſcending in a fluid is equal to that which a heavy body would acquire in falling through a ſpace equal to $\frac{4}{3}$ of its diameter, multiplied by the difference between the numbers expreſſing the ſpecific gravities of the globe and fluid.

For $n-1$ is the relative gravity of the globe, and $\frac{1}{6}pd^3$ its magnitude, therefore $\frac{1}{6}pd^3 \times \overline{n-1}$ is the relative weight by which it is urged downwards; and the reſiſtance $= \frac{pd^2z}{8}$, now when the velocity is greateſt $\frac{pd^2z}{8} = \frac{1}{6}pd^3 \times \overline{n-1}$, or $z = \frac{4}{3}d \times \overline{n-1}$.

16. Theſe uniform velocities will be in the ſubduplicate ratio of the diameters of the bodies.

For z or the ſpace through which a body muſt fall by the force of gravity to acquire the greateſt velocity, is equal $\frac{4}{3}d \times \overline{n-1}$, and therefore is directly as d; but the velocity is in the ſubduplicate ratio of the ſpace.

17. Mathematically ſpeaking, bodies deſcending in fluids will not acquire their ultimate and uniform velocity in any finite time whatever.

Fig. 31. The abſolute force with which the body deſcends is the difference between its weight and the weight of an equal bulk of the fluid; and this difference divided by the weight of the body will be the accelerating force, which let

let us suppose equal to d; now the resisting force increases as the square of the velocity v, and therefore will be equal to some constant quantity multiplied into v^2; let this quantity be e, therefore the absolute accelerating force, upon the whole, will be $d-ev^2$. Let the constant force d be represented by the given line AC, and let the decrement of this force by the resistance, be AK, and consequently the absolute accelerating force $= KC$; also let the absolute velocity AP of the body be a mean proportional between AC and AK, and therefore in the subduplicate ratio of AK. Let the increment of the resisting force be KL, and the contemporaneous increment of the velocity be PQ; with the centre C, and the rectangular asymptotes AC, CH, let the hyperbola BNS be described, meeting the ordinates AB, KN, LO. Because $AK :: AP^2$, the moment of the former KL will be as the increment $2APQ$ of the latter; that is, as $AP \times KC$, for the increment PQ of the velocity is as the absolute accelerating force KC; therefore $KL \times KN :: AP \times KC \times KN :: AP$, because $KC \times KN$ is constant. Therefore the indefinitely little hyperbolic area $KNOL :: AP$. And the hyperbolic area $ABOL$ is composed of the particles $KNOL$ always proportional to the space described with that velocity, the particle of time, in which KL is generated, being given. Consequently, when KC the absolute accelerating force vanishes, that is, when the motion of the body becomes uniform, the space described $ABSHCA$, and therefore the time, will be infinite.

18. The

18. The retardations of ſpherical bodies moving in different fluids are to each other as the denſities of the fluids and ſquares of the velocities directly, and as the denſities of the bodies and their diameters inverſely.

We muſt take care to diſtinguiſh the reſiſtance of a body from its retardation; reſiſtance is the quantity of motion, retardation the quantity of velocity, which is loſt; therefore the retardations are as the reſiſtances applied to the quantities of matter; and in the ſame body the reſiſtances and retardations are proportional to each other.

19. If the fluid does not flow in behind the body as faſt as the body moves forward, the reſiſtance will very much depend on the configuration of the hinder part of the body.

Thus the reſiſtance to the fore-part of a cylinder is leſs than on the equal flat ſurface of a cone, or of a hemiſphere. See Hutton's Dict. Article Reſiſtance.

LECTURE

LECTURE XXVII.

1. FRICTION is the reſiſtance which a moving body meets with from the ſurface on which it moves; and may be conſidered as a retarding force, acting in a direction contrary to that which moves the body.

2. If an hard flat body be drawn along another, its friction will be an uniformly retarding force.

Hence it follows, that the quantity of friction does not depend on the velocity with which the rubbing parts move upon each other. See Vince's Lectures.

3. If M, conſidered as an hanging weight, be the moving force, F the quantity of friction conſidered as equivalent to a weight without inertia, drawing the body back upon an horizontal plane, S the ſpace deſcribed by M in t ſeconds, and

$g = 193$ inches, then $S = \frac{M - F}{M + W} \times gt^2$; whence $F = M - \frac{M + W}{gt^2}$.

See Vince's Lectures.

4. The quantity of friction encreafes in a lefs ratio than the weight of the body.

If M and W be encreafed in any ratio, then if F increafed in the fame ratio, the fame fpace would be defcribed in the fame time; but by thus increafing M and W, it appears, by experiment, that S is increafed in the fame time; therefore F muft have increafed in a lefs ratio.

5. The friction of a body increafes with the furface, but in a lefs ratio.

6. A greater force is requifite to fet a body in motion than to continue that motion without acceleration.

By a variety of experiments it is found, that about $\frac{1}{3}$d of a weight is requifite to move that weight, when drawn along an horizontal furface; and about $\frac{1}{4}$th of the weight to continue it in motion. This difference has, in general, been attributed to the attraction of cohefion; but that this attraction cannot be the caufe, appears from this, that the difference of the forces neceffary to fet a body in motion, and to continue it, which is generally about $\frac{1}{12}$ of the whole weight of the body, is much lefs than the force fufficient to overcome the attraction of cohefion even in a direction perpendicular to the furfaces. The caufe of the difference therefore is probably the momentum of the body

body once set in motion, by which it happens that the moving body, in descending down the face of the little cavities in the plane on which it moves, has a tendency to ascend the face of the next cavity.

7. If a body whose line of direction falls within the base, be drawn along an horizontal plane by a force acting at the centre of gravity, and parallel to the plane, and the friction be equal to or greater than the moving force, the body will roll without sliding, when the weight is to the moving force in a less ratio than radius to the tangent of the angle which the vertical line makes with that drawn from the centre of gravity to the centre of rotation.

Fig. 32. Let C be the centre of gravity, CB the line of direction, Cc the direction of the moving force parallel to the plane SO; since the friction is equal to or greater than the moving force, the point A will be at rest, and AC may be considered as a lever whose fulcrum is A, and the extremity C acted on by two forces, the moving force in the direction Cc, and the weight of the body in the direction CB; now the efficacy of the moving force to turn the lever is as the force $\times$ sine of the angle cCA; and the force of the weight to move the lever in the contrary direction, is as the weight $\times$ sine of the angle BCA; therefore these forces are equal when the moving force $=$ weight $\times \frac{\text{sin.}}{\text{cos.}} BCA =$ weight $\times$ tangent BCA. In a sphere, since the tangent of BCA vanishes, it will roll, whatever be the magnitude of the moving force.

 8. If

8. If the friction be lefs than the moving force, the body will both flide and roll, when the weight is to the difference between the friction and the moving force in a lefs ratio than radius to the fame tangent.

For the point *A* is moved forward by the difference between the moving force and friction; therefore in order to determine the motion of the body we may confider *A* at reft, and *C* acted on in the direction *Cc* with a force equal to that difference.

9. If the friction be lefs than the moving force, the body will flide without rolling, when the weight is to the difference between the moving force and friction, in the fame or a greater ratio than radius to the tangent of the fame angle,

In a fphere this can never be the cafe, if it be of any finite weight.

10. If the friction vanifh, the body will flide without rolling, whatever be its weight or moving force.

11. When the line of direction falls without the bafe, if the friction be equal to or greater than the moving force, the body will roll without fliding.

12. If the friction be lefs than the moving force, the body will both flide and roll.

13. If a body be placed on an inclined plane, and the plane be elevated until the body will flide, if the elevation be further increafed by the fmalleft

ſmalleſt quantity; then will the friction on the inclined plane be to the weight of the body, as the ſine of the angle of elevation to radius.

The angle of elevation in this caſe is called the angle of Quieſcence.

14. The friction on an horizontal plane is to the weight of the body, as the tangent of the angle of quieſcence to radius.

For the friction on the horizontal plane is to the friction on the inclined plane, as radius to the coſine of the angle of quieſcence.

15. When a body deſcends down an inclined plane, the phænomena of ſliding and rolling are determined in the ſame manner as on an horizontal plane, if inſtead of the weight we ſubſtitute the weight × coſine of the angle of elevation; and inſtead of the vertical line, a line perpendicular to the inclined plane.

16. When a ſphere rolls down an inclined plane without ſliding, the velocity with which any point in the periphery revolves round the centre, is equal to that with which the centre of gravity moves.

For when the ſphere has performed one revolution, the centre of gravity has deſcribed a ſpace equal to the circumference, therefore the ſurface and the centre of gravity move with equal velocities.

17. When

17. When a ſphere rolls down an inclined plane, the accelerating force is to the force which would accelerate it when ſliding without friction, as 5 to 7.

Let r be the radius of the ſphere, g the diſtance of the centre of gyration from the centre, w the weight of the ſphere, h the height of the plane, and l its length; the inertia of the ſphere which reſiſts the communication of motion to the circumference, to which the moving force is applied, will be the ſame as if $w \times \frac{g^2}{r^2}$ were collected in the circumference (Lect. 19. Art. 22.), ſince it moves equally faſt with the centre of gravity; the whole maſs moved is therefore $w + \frac{w \times g^2}{r^2} = \frac{7}{5} w$, becauſe $g^2 = \frac{2}{5} r^2$, but the moving force is the ſame in both caſes, whether there be friction or not; therefore the accelerating forces will be inverſely as the quantities of matter moved.

18. A power which moves a body along an horizontal plane, acts with the greateſt advantage, when the line of direction makes an angle of 18° with the plane.

Fig. 33. Let A be the body which is to be moved along the horizontal plane AC, by a given power eſtimated in quantity and direction by AB; let fall the perpendicular BC; let the given line $AB = 1$, $BC = x$, $AC = \overline{1 - x^2}|^{\frac{1}{2}}$ = the force moving the body horizontally. The power, by its oblique action, diminiſhes the preſſure of the weight

weight on the horizontal plane, in the ratio of $1 : x$; therefore $Ax =$ that part of the pressure which is taken off, and the actual pressure $= A - Ax$. Let friction $= \frac{m^{\text{th}}}{n}$ part of the weight; in this case it will be $= \frac{m}{n} A - \frac{m}{n} Ax$. The force therefore requisite to move A horizontally, must be equal to the horizontal force diminished by friction $= A \times \overline{1-x^2}|^{\frac{1}{2}} - \frac{m}{n} A + \frac{m}{n} Ax$, a minimum, therefore its fluxion $\frac{m}{n} A \dot{x} - \frac{Ax\dot{x}}{\overline{1-x^2}|^{\frac{1}{2}}} = o$, and $x = \frac{m}{\overline{m^2+n^2}|^{\frac{1}{2}}} =$ sine of the angle BAC; which, since $\frac{m}{n} = \frac{1}{3}$ in general, will be $= \frac{1}{\sqrt{10}}$.

19. If the plane, along which the body is to be moved, be inclined to the horizon, the sine of the angle which the line of direction of the power makes with the plane, when it acts with the greatest advantage will be $= \frac{c}{c^2+9}$, c being the cosine of the angle of elevation to radius $=$ unity.

20. Friction is diminished by making the surfaces smooth which move upon each other.

But there is a limit to this smoothness, for the polish of the surfaces may be so far increased, as to render the attraction of cohesion very sensible.

21. Friction

21. Friction is diminiſhed by anointing the rubbing ſurfaces with ſome unctuous matter.

22. Friction is diminiſhed by cauſing ſubſtances of different kinds to move upon each other.

Both becauſe the attraction of coheſion will be leſs, and alſo becauſe, on account of the diſſimilarity of the ſurfaces, the eminences and cavities will not ſo often correſpond.

23. Friction is diminiſhed by diminiſhing the ſurfaces in contact.

24. Friction is diminiſhed by cauſing the body to roll rather than ſlide along the ſurface.

25. The effect of friction is diminiſhed by diſpoſing the parts of the machine in ſuch a manner, as that the ratio of the velocity of the parts which rub againſt each other to the velocity of the power, may be as ſmall as poſſible.

LECTURE

LECTURE XXVIII.

1. THE advantage of wheels arises from their turning on their axles.

For when a wheel turns on an axle, the force to overcome the friction is diminished in the ratio of the radius of the wheel to the radius of the axle; and no advantage is gained if they do not turn.

2. A wheel carriage is drawn by the least power, when the line of draught passes through the centre of gravity of the carriage, and parallel to the plane on which it moves; or rather when it forms a small angle with it.

3. If the wheels be all equal, it requires the same force to draw the carriage, whether it be loaded before or behind.

If the plane be hard, it is evident that the breadth of the wheels will make no difference in the result.

4. A carriage is more eaſily drawn with four equal wheels, than when the two fore wheels are ſmaller than the two hind wheels.

5. If two wheels be high and two low, it makes no difference which go before.

However it is convenient that the fore wheels ſhould be the lower, 1st. Becauſe leſs room is requiſite for the turning of the carriage. 2dly. A horſe is generally the moving power, and from the make of his body, he draws with moſt advantage when the traces make a ſmall angle with the plane on which he moves. 3dly. A part of the force of the horſe will be thus exerted in lifting the weight as well as in drawing it; and it is found that a horſe moves a greater weight, when a part of it is laid on his back. 4thly. In moving over obſtacles, the power will be exerted more nearly in a direction parallel to the plane on which the body is then moving. See Wallis vol. 1. p. 979.

6. If the wheels be all equal, it requires a greater weight to draw the carriage when they are leſs than when greater.

Becauſe the reſiſtance of the ground, which turns the wheels about, more eaſily overcomes the friction at the axle in a large than in a ſmall wheel.

7. If the fore wheels of a carriage be leſs than the hind wheels, it will be drawn upon an horizontal plane with greater eaſe when the load is placed behind, than when before.

Fig. 3.

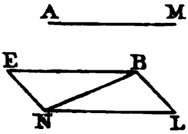


Fig. 2.

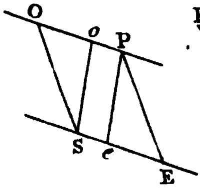
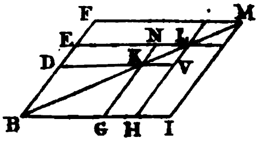


Fig. 1.



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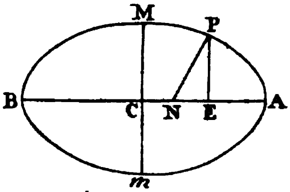

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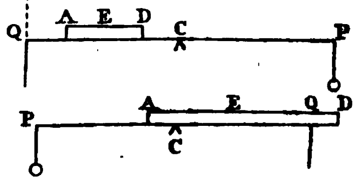
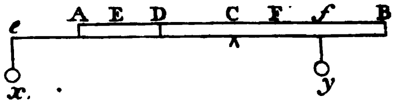


Fig. 6.



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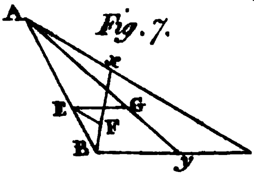


Fig. 8.

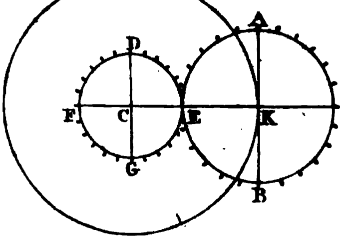
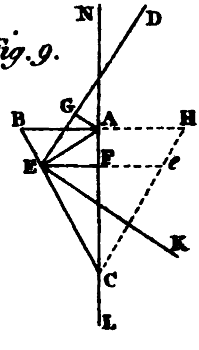


Fig. 9.



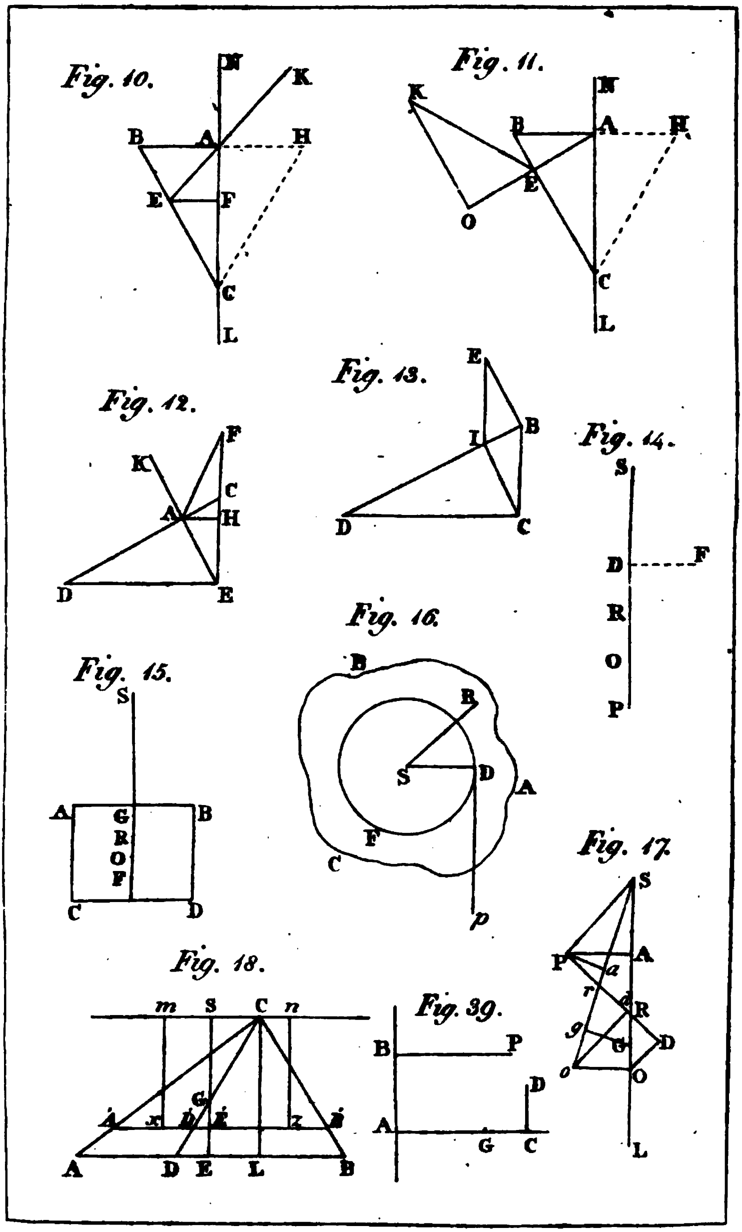


Fig. 10.
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Fig. 39.

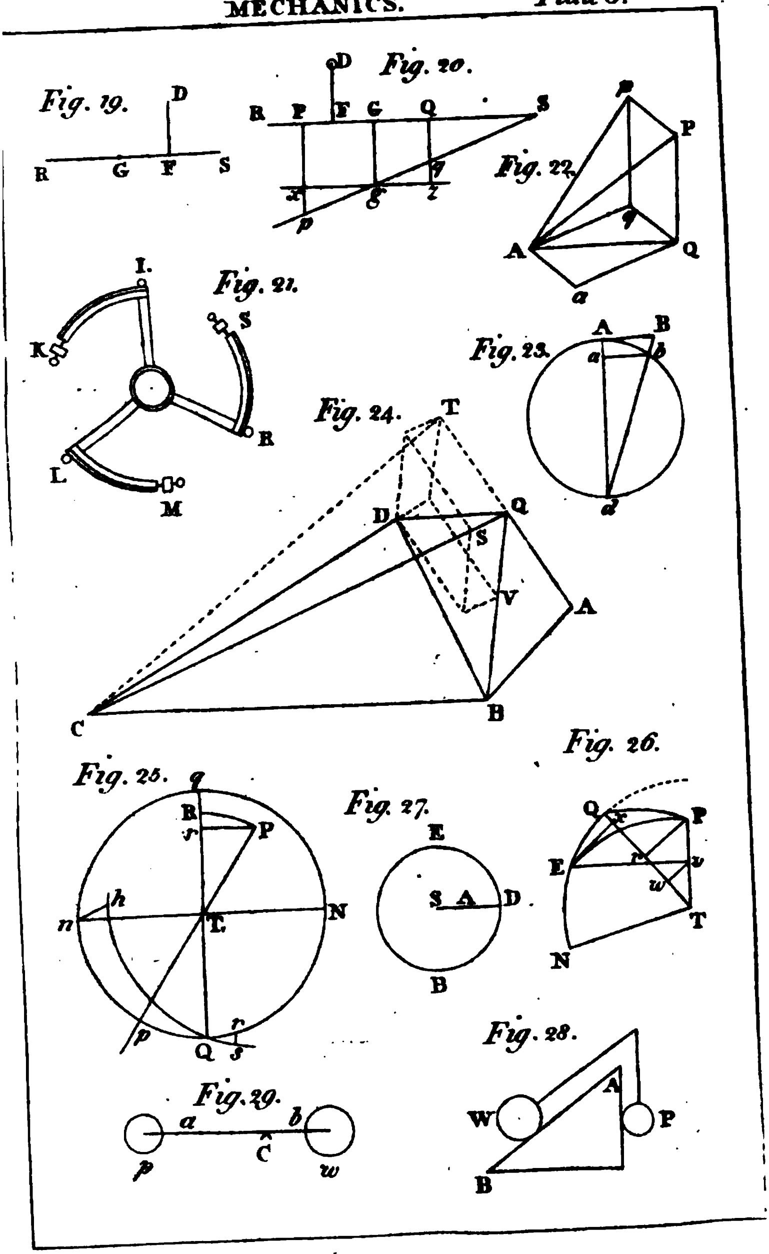
Fig. 19.
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Fig. 30.

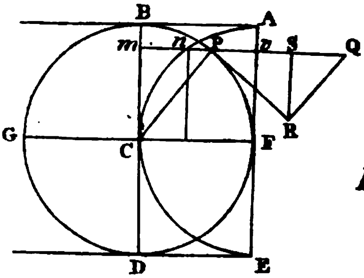


Fig. 31.

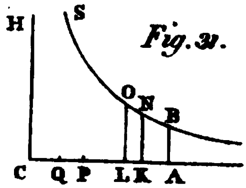


Fig. 32.

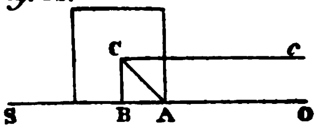
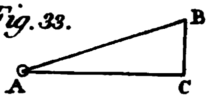


Fig. 33.



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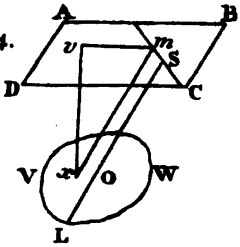


Fig. 35.

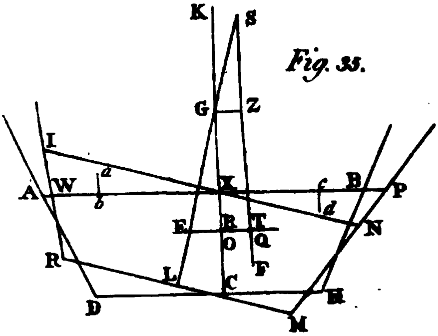


Fig. 36.

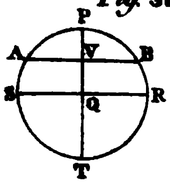


Fig. 38.

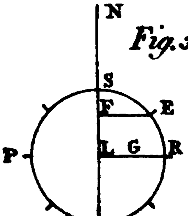
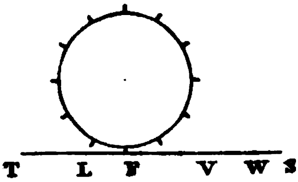


Fig. 37.



HYDROSTATICS.

LECTURE I.

1. HYDROSTATICS is that part of Natural Philofophy, which treats of the preffure and equilibrium of fluids.

2. Fluids are divided into two claffes, coherent and incoherent.

They are fometimes otherwife diftinguifhed into compreffible and incompreffible; but this diftinction, ftrictly fpeaking, appears to be inaccurate: for all bodies being porous, their parts may be brought nearer to each other; and a liquid is an affemblage of folid bodies, and therefore fhould alfo be compreffible. Accordingly Mr. Canton has proved that water is compreffible.

3. Fluids gravitate in their own element, or, as it is otherwife expreffed, in proprio loco.

For when a body fpecifically lighter than the fluid floats on it, if we fuppofe an imaginary plane to touch the

the lower furface of the folid, every part of this plane muft be equally preffed, otherwife a motion would enfue; but the plate of water on which the folid refts, fuftains the whole weight of the folid; therefore every other equal plate at the fame depth muft fuftain an equal preffure; now it is found that the columns of the fluid, which are infiftent on equal plates of water at the fame depth, are of equal weight with the folid.

4. A lighter fluid gravitates on an heavier, and an heavier on a lighter.

5. The preffure of fluids is propagated equally in all directions.

6. If a fluid be ftagnant, the direction of the preffure is perpendicular to the tangents in the feveral points of the furface.

7. The furface of every ftagnant liquid is horizontal.

8. On the earth, the furface of a fluid of fmall extent may be confidered as a plane.

9. If the parts of a fluid gravitate towards a common centre, the furface of the fluid, when in equilibrium, will be fpherical.

10. If a fluid mafs be in equilibrium, the fluid in any communicating canals will alfo be in equilibrium.

11. A fluid mafs divided into various laminæ, whofe tangents are perpendicular to the direction of gravity, will be in equilibrium, if in each point the

the thickneſs of the ſtratum be reciprocally as the force of gravity in that point.

12. If the denſity of a fluid be uniform, the preſſure at any depth is in proportion to the depth.

13. Fluidity does not conſiſt in the inteſtine motion of its parts.

14. Fluidity, according to the Newtonian ſyſtem, ariſes from a want of any ſenſible coheſion between the conſtituent particles of the fluid.

This want of coheſion is attributed to the ſpherical figure of the particles: for though the contacts of ſpheres are merely points, yet ſince the force of coheſion extends to ſome finite diſtance, and that diſtance is the ſame in all caſes, the ſpheres will attract each other by the action of circles whoſe centres are the points of contact, and whoſe radii are = diameter ½ × ſquare root of the ſpace through which the attraction extends; but theſe circles are as the ſquares of their radii, that is, directly as the diameters of the ſpheres; and therefore if the ſpheres be nearly evaneſcent, the attraction of coheſion, which is proportional to them, will be likewiſe phyſically evaneſcent.

15. Fluids of uniform denſity preſs according to their perpendicular altitudes, whatever be their quantities, or the ſhape of the containing veſſels.

16. In communicating veſſels, a given fluid will ſtand at the ſame height, whatever be their forms or poſition.

17. If

17. If two immiſcible fluids be poured, the one on the other, their common ſurface will be level.

18. If two immiſcible fluids communicate, they will ſtand at altitudes from the plane, where they meet, which are inverſely as their ſpecific gravities.

LECTURE

LECTURE II.

1. THE preſſure ſuſtained by any ſurface is equal to the weight of a pillar of the liquid, whoſe baſe is the preſſed ſurface, and height is the perpendicular diſtance of the centre of gravity of that ſurface from the upper ſurface of the liquid.

2. A body immerſed in a fluid is preſſed by it in every direction, and the preſſure is greater in proportion to the depth to which it is immerſed.

Hence is derived a method of meaſuring the depth of an unfathomable ſea: to the bottom of a hollow ſphere is attached a veſſel containing ſome mercury, in which is immerſed a tube, hermetically ſealed at top, and within this tube is a little floating iron gage, which by the preſſure of a very tender ſpring, adheres to whatever part of the tube it is forced; in the upper part of the veſſel are ſome holes to admit

admit the water; to the bottom of this veſſel is added a heavy weight, which is immediately diſengaged, when the inſtrument ſtrikes the bottom; it then aſcends, and the height to which the mercury has been made to aſcend, by the preſſure of the water, indicates the depth to which the inſtrument had deſcended.

3. The centre of preſſure of a ſurface is that point, to which if a force equal to the whole preſſure were applied, but in a contrary direction, it would keep the ſurface at reſt.

4. If a plane ſurface which is preſſed by a liquid be produced to the ſurface of the liquid, and their common interſection be made the axis of ſuſpenſion, the centre of percuſſion will be the centre of preſſure.

Fig. 34. Let $ABCD$ be the ſurface of the fluid, which preſſes on the plane VW; produce the plane until it meets the fluid in the line mS; let O be the centre of preſſure; from any point x, of the ſurface, erect the perpendicular xv to the ſurface of the fluid, and in the ſurface draw vm perpendicular to the interſection mS. The preſſure on x is as $x \times xv$; and its effect to turn the plane about mS is as $x \times xv \times mS$; alſo its effect to turn the plane about SL, will be as $x \times xv \times mS$. In like manner if the plane VW be ſuppoſed to revolve round the axis mS, and to ſtrike an obſtacle at O, the percuſſive force of the particle x, by which it endeavours to move the plane about mS, will be as $x \times xm^2$, or as $x \times xm \times xv$; and its force to turn the plane

plane about SL, will be as $x \times xm \times mS$, or as $x \times xv \times mS$. The percussive forces therefore of the particles, whereby they endeavour to move the plane in the two directions, are the same with the forces of pressure, and therefore the centres of pressure and percussion are coincident.

5. The centre of pressure for a right line or rectangular plane surface is at the distance of two-third parts from the surface of the fluid.

6. If AB be the intersection of the pressed surface and of the surface of the fluid, AC a plane passing through the centre of gravity and perpendicular to AB, DC a perpendicular let fall from any particle of the pressed surface on the plane AC, and P the centre of pressure, from which is drawn PB perpendicular to AB, then will AB, the distance of the centre of pressure from the plane AC, be equal to the sum of all the $\frac{GC \times CD \times D}{\text{Body} \times AG}$.

Fig. 39. For the sum of all the forces with which the surface is turned in one direction round PB as an axis $=$ sum of all the $D \times AC \times \overline{AB - DC}$; and the sum of all the forces with which it is turned in the contrary direction $=$ sum of all the $D \times AC \times \overline{DC - AB}$; therefore all the $D \times AB \times AC$ $=$ all the $D \times DC \times AC$; but all the $D \times AB \times AC =$ all the $D \times AB \times AG +$ all the $D \times AB \times GC$; and all the $D \times AB \times GC = 0$, from the nature of the centre of gravity: in the same manner, all the $D \times DC \times AC =$ all the $D \times DC \times GC +$ all the $D \times AG \times DC$; but all the

$D \times AG \times DC = 0$; therefore all the $D \times AB \times AG =$ all the $D \times GC \times DC$; and $AB = \frac{\text{all the } D \times GC \times DC}{\text{all the } D \times AG}$.

The ſame rule ſerves for determining the centre of percuſſion; See Phil. Trans. No. 345, page 339.

Since the centre of oſcillation always lies in the right line, paſſing through the centre of gravity and perpendicular to the axis of motion, or interſection of the preſſed ſurface and ſurface of the fluid, it follows that the centre of preſſure is not always coincident with the centre of oſcillation.

7. The entire lateral preſſure of a veſſel whoſe ſides are perpendicular to the baſe, is equal to the weight of the fluid contained in a triangular priſm whoſe altitude is that of the fluid, and baſe is a parallelogram, one ſide of which is equal to the altitude of the fluid, and the other to the perimeter of the veſſel.

8. If a tube open at both ends, be inſerted into a cloſed veſſel, and water be poured in, ſo as to riſe in the tube to any altitude, the preſſure which the top of the veſſel ſuſtains, is equal to the weight of a pillar of the liquid, whoſe baſe is the area of the lid of the veſſel, and altitude equal to that of the water in the tube.

Hence is explained the Hydroſtatical paradox, that the greateſt weight may be ſuſtained by the leaſt aſſignable quantity of water.

9. If

9. If a body be immerſed in a liquid of a leſs or a greater ſpecific gravity, the force with which it deſcends or aſcends is the difference between the weight of the ſolid and an equal bulk of the liquid.

The force with which a body ſpecifically lighter than a fluid, endeavours to aſcend in it, may be applied to raiſe heavier bodies, namely by uniting them firmly together. On this principle are conſtructed the machines which in Holland are called *Camels.*

10. If a body be immerſed in a fluid ſpecifically lighter, it loſes a portion of its weight equal to that of an equal bulk of the ſolid.

11. The weights loſt by different bodies are as their magnitudes, whatever be their ſpecific gravities.

12. This weight is not abſolutely loſt, but is added to the fluid.

13 This weight is not varied either by the depth of the fluid, or the depth to which the body is immerſed.

LECTURE III.

1. THE denſities or ſpecific gravities of bodies are directly as their weights, and inverſely as their magnitudes.

In eſtimating the weights, magnitudes, and ſpecific gravities of bodies, ſome ſtandard quantities are always aſſumed, to which others are referred; the ſpecific gravity and weight of a given magnitude of water are thoſe ſtandard quantities, the denſity of rain water being found more uniformly the ſame than that of any other ſubſtance.

2. The ſpecific gravity of a ſolid is to that of a lighter fluid, as the abſolute weight of the ſolid to the weight which it loſes when immerſed in the fluid.

3. The ſpecific gravities of two ſolids both ſpecifically heavier than a fluid, are to each other directly

rectly as their absolute weights, and inversely as the weights which they lose in the fluid.

If the absolute weights be equal, the specific gravities will be reciprocally as the weights lost; therefore if two bodies of different specific gravities balance each other in air, when immersed in water the equilibrium will be destroyed. Thus if two cylinders of copper and brass be exactly counterpoised in air, and both be immersed in water, the copper will preponderate. The weight requisite to restore the equilibrium may be thus found; let w be the common weight of the brass and copper, b the specific gravity of brass, and c the specific gravity of copper, that of water being unity; then the weight lost by the brass $=$ $\frac{w}{b}$; for the same reason, the weight lost by the copper $=$ $\frac{w}{c}$; and the difference of the weights lost $= \frac{w}{b} - \frac{w}{c} =$ $\frac{c-b}{cb} \times w$.

4. The specific gravities of two fluids both specifically lighter than a solid, are to each other directly as the weights which that solid loses, when immersed in the fluids respectively.

A cubic inch of any substance being immersed in water loses 253.18 grains of its weight; if it be immersed in spirits of wine it loses 217 grains, therefore the weight of a cubic inch of water is 253.18 grains, and that of a cubic inch of spirits of wine is 217 grains; wherefore the specific

ſpecific gravities of water and ſpirits of wine are as 253.18 : 217; or as 1 : ,857.

5. If a ſolid ſpecifically heavier than a fluid be immerſed to a depth, which is to its thickneſs, as the ſpecific gravity of the ſolid to that of the fluid, and the preſſure of the fluid from above be removed, the body will be ſuſtained by the fluid.

6. If a body ſpecifically lighter than a fluid be immerſed therein, and the preſſure from beneath be removed, the body will continue depreſſed.

Hence it follows, that there is no ſuch property in matter as abſolute levity, by which bodies are made to aſcend from the ſurface of the earth, in the ſame manner as they are made to deſcend towards it by the force of gravity. If cork aſcended in mercury by an abſolute levity, it ought alſo to aſcend, even though the preſſure of the mercury upwards were removed, ſince removing the mercury from beneath could not deſtroy any eſſential property belonging to the cork itſelf. When bodies therefore aſcend in fluids, they aſcend by their comparative, not by their abſolute levity; that is, they aſcend becauſe they are lighter than the fluid, and not becauſe they have a tendency contrary to that of gravity.

7. If a body be of variable ſpecific gravity, it will either ſink, aſcend, or remain ſuſpended, according to the degree of that variable ſpecific gravity at different periods.

LECTURE IV.

1. IF a body float on the furface of a fluid, it difplaces as much of the fluid as is equal in weight to the whole body.

2. A folid floats permanently in a given pofition, when the fmalleft inclination from that pofition being produced, the line of fupport is moved towards thofe parts which are immerfed by the inclination.

The line of fupport is the vertical line paffing through the centre of gravity of the part immerfed,

3. A body cannot float permanently on a fluid, unlefs the centres of gravity of the folid and of the part immerfed be fituated in the fame vertical line.

4. A body will not always float permanently when

when theſe centres are ſituated in the ſame vertical line.

5. If the line of ſupport be moved towards thoſe parts which are elevated by the inclination, the ſolid will overſet.

6. If the line of ſupport, notwithſtanding the inclination of the body, ſtill paſs through the centre of gravity, the ſolid will continue at reſt in any poſition.

This, as is eaſily ſeen, will take place in a floating ſphere, in which the centre of gravity of the part immerſed is always in the ſame vertical line.

7. When a ſolid is placed on the ſurface of a fluid, ſo that the centres of gravity of the ſolid and part immerſed ſhall be in the ſame vertical line, the ſolid is ſaid to be in a ſtate of equilibrium.

From the preceding obſervations it appears, that there are three ſpecies of equilibrium, viz. of ſtability, inſtability, and indifference.

8. When a floating body revolves round a given axis, and paſſes through ſeveral poſitions of equilibrium, thoſe of ſtability and inſtability are alternate.

For if we ſuppoſe the body to revolve on its axis from one poſition of ſtable equilibrium to the next, there muſt be ſome intermediate poſition, in which it neither inclines to the one nor to the other; this therefore muſt be a poſition of unſtable

unſtable equilibrium; becauſe if the body be inclined either to the one ſide or the other, it will endeavour to regain either the one poſition of ſtability, or the other.

9. The angular motion of a floating body is round its centre of gravity.

Any force being given which acts upon a body, in order to determine its angular motion, it is to be reſolved into two forces, one of which paſſes through the centre of gravity, and produces a progreſſive motion only; the other, which is at right angles to the former, is entirely employed in generating an angular motion round that centre.

10. If a ſolid be placed on a fluid, and there be let fall a perpendicular from the centre of gravity of the part immerſed on the horizontal ſurface of the fluid, and on this perpendicular there be let fall another from the centre of gravity of the whole ſolid, and a plane be drawn paſſing through theſe two perpendiculars; when the ſolid is diſturbed from its equilibrium, it will begin to revolve round an axis which paſſes through the centre of gravity of the body, and is perpendicular to the before-mentioned plane.

11. The force of rotation is proportional to the weight of the body, multiplied by the perpendicular diſtance between the two vertical lines paſſing through the centres of gravity of the whole ſolid and the part immerſed.

12. If AB be the water line of a floating body whose figure is uniform with respect to the axis of motion, V the volume of the part immersed, P the part immersed in consequence of the inclination of the solid from its state of equilibrium through an evanescent angle whose sine is s, radius being 1, G the centre of gravity of the solid, O the centre of gravity of the part immersed, then will GZ, the perpendicular distance between the two vertical lines passing through the centres of gravity of the part immersed and the whole body, $=$
$\frac{2}{3}\frac{AB \times P}{V} - GO \times s$.

Fig. 35. Let $ABHD$ be the part immersed of a floating body whose centre of gravity is G, O the centre of gravity of the part immersed; let the body be inclined round G through the evanescent angle SGK so as to assume the position $INMR$; take $GE = GO$, and from E let fall the perpendicular ET on OG the vertical line passing through G; the part XPN which is immersed in consequence of the inclination $= IXW$ the part elevated above the surface, and the vertical angles at X are equal, therefore the intersection of the lines IN and AB, that is, X will bisect the line AB, and the points P, B, N will coincide; let d and a be the centres of gravity of these parts, from which let fall the perpendiculars ab, dc on AB. Since the part IXW which was before immersed, is now elevated, and XPN equal to it is depressed, the former area has been subducted from the one

side

side and transferred to the other, and its centre of gravity has been moved horizontally through the space bc, therefore the centre of gravity of the whole solid will be moved in the same direction; suppose it had been moved to Q, through Q draw the vertical line QS meeting ET in T, then as $V : P :: (bc = \frac{4}{3}XB =) \frac{2}{3}AB : ET$ (Art. 11. L. 11.) $= \frac{2}{3}AB \times \frac{P}{V}$. Also $1 : s :: GE = GO : ER = GO \times s$, whence RT or $GZ = ET - ER = \frac{2}{3}AB \times \frac{P}{V} - GO \times s$.

We are to observe, that the centre of gravity E is always, of course, moved in the direction ET, when the body is depressed on that side, because there is an increase of the depressed part on that side, and a deduction on the other; on the quantity of this translation depend the three species of equilibrium. If the quantity of the translation of E be so great as to pass the line GO, as represented in the figure, that is, if ER be less than ET, the equilibrium will be that of stability; if the translation be such, as that ER shall be equal to ET, or $GO \times s = \frac{2}{3}AB \times \frac{P}{V}$, the force of rotation will be evanescent, and the equilibrium will be that of indifference; and if the translation be so little, as that ET shall be less than ER, or that $\frac{2}{3}AB \times \frac{P}{V} - GO \times s$ shall be a negative quantity, the equilibrium will be that of instability, and the body being inclined through the evanescent angle KGS will overset.

 13. Also

13. Alſo the ſtability of the ſolid, which is proportional to GZ, will be as $\frac{AB^3 \times s}{12\,V} - GO \times s$

For the evaneſcent area $P = NXP = \frac{XB^2 \times s}{2} = \frac{AB^2 \times s}{8}$, which being ſubſtituted, we have $GZ = \frac{AB^3 \times s}{12\,V} - GO \times s$

14. If the ſolid be of an irregular form, the ſtability will be as the ſum of all the $\frac{AB^3 \times s}{12\,V} - GO \times s$, AB repreſenting the water line of every vertical ſection of the ſolid, perpendicular to the axis of motion.

15. If the floating body be an oblong rectangular parallelepiped, whoſe altitude is perpendicular to the ſurface of the fluid, the ſtability of this ſolid will be proportional to the difference between the ſixth part of the ſquare of the breadth of the baſe, and the product of the ſquare of the altitude into the difference between the number expreſſing the ſpecific gravity of the ſolid and its ſquare, that of the fluid being unity.

Let a be the altitude of the ſolid, b the breadth of the baſe, and n the ſpecific gravity of the ſolid, AB^3 the cube of the water line $= b^3$, the height of the part immerſed $= na$, and therefore V the part immerſed $= nab$; GO the diſtance

tance between the centres of gravity of the whole body and the part immerſed $= \frac{1}{2}a - \frac{1}{2}na = \frac{a-na}{2}$, therefore the ſtability :: $\frac{b^3}{12nab} - \frac{a-na}{2} = \frac{b^2}{6} - a^2 \times \overline{n-n^2}$.

Thus for example, ſuppoſe the height to be equal to the baſe, or $a = b$, and it be required to aſcertain the ſpecific gravity of the ſolid, when it will float in the inſenſible equilibrium; here $\frac{b^2}{6} = a^2 \times \overline{n - n^2}$, therefore $n^2 - n = -\frac{1}{6}$; and $n = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{6}}$, that is, the ſpecific gravity of the ſolid may be either .78868, or .21132.

16. If the floating body be a cylinder, whoſe axis is vertical, the ſtability of the cylinder will be as the quotient of the ſquare of the radius of the cylinder, divided by four times the height of the part immerſed, diminiſhed by the diſtance between the centres of gravity of the whole ſolid and the part immerſed,

Fig. 36. Let *RPST* be the ſection of the cylinder made by the fluid, *AB* the water line of a ſection perpendicular to the axis of motion, $RQ = r$, and $QV = x$; then $AB = 2\sqrt{r^2-x^2}$, and $AB^3 = 8\overline{r^2-x^2}|^{\frac{3}{2}}$; but $\overline{r^2-x^2}|^{\frac{3}{2}} = \overline{r^2-x^2}|^{\frac{1}{2}} \times r^2 - \overline{r^2-x^2}|^{\frac{1}{2}} \times x^2$. Now $\overline{r^2-x^2}|^{\frac{1}{2}} \times r^2 = r^2 \times \frac{1}{4}$ of the circle whoſe radius is r, when x increaſes from o to r; and $\overline{r^2-x^2}|^{\frac{1}{2}} \times x^2 = rx^2 - \frac{x^4}{2r} - \frac{x^6}{8r^3}$, &c.

and

and the quantity generated $=\frac{rx^3}{3}-\frac{x^5}{10r}-\frac{x^7}{56r^3}$, &c. See *Anal. per Equationes*, &c. which, when x increases from o to r, becomes $r^4 \times : \frac{1}{3}-\frac{1}{10}-\frac{1}{56}$, &c. $= r^4 \times \frac{1}{4}$ of a circle whose diameter is 1, (vide *Anal. per Equationes*, page 74.) $= r^2 \times \frac{1}{16}$ of a circle whose radius is r; therefore the whole quantity generated by $\overline{r^2-x^2}|^{\frac{3}{2}}$ will be $r^2 \times \overline{\frac{1}{4}-\frac{1}{16}}$ of a circle whose radius is r, that is, $= r^2 \times \frac{3}{16}$ of that circle; and the sum of all the AB^3 in one semi-circle $= 8r^2 \times \frac{3}{16}$ of the circle $RPST$, and the sum of all the AB^3 in both semi-circles $= 3r^2 \times$ area of the circle $RPST$; the volume of the part immersed is equal to its depth (d) $\times$ the circle $RPST$, therefore $\frac{AB^3 \times s}{12V} = \frac{3RQ^2 \times \text{circle } RPST \times s}{12d \times \text{circle } RPST}$

$$= \frac{RQ^2 \times s}{4d}; \text{ and } \frac{AB^3 \times s}{12V} - GO \times s = \frac{RQ^2}{4d} - GO \times s.$$

If n be to l as the specific gravity of the solid to that of the fluid, then the depth of the part immersed will be ln, and the whole height of the cylinder will be l, therefore $GO = \frac{1}{2}l - \frac{1}{2}ln = \frac{l-ln}{2}$; and the stability of the cylinder will be proportional to $\frac{r^2}{4ln}-\frac{l-ln}{2}$; and will vanish when $\frac{r^2}{4ln}$ $= \frac{l-ln}{2}$, or $n^2 - n = -\frac{r^2}{2l^2}$, or $n = \frac{1}{2} \pm \sqrt{\frac{1}{4}-\frac{r^2}{2l^2}}$; or if b be put equal to the diameter of the base, $n = \frac{1}{2} \pm \sqrt{\frac{1}{4}-\frac{b^2}{8l^2}}$. Therefore, if the specific gravity of the cylinder

linder be half that of the fluid, and its height half the diameter of the base, the cylinder will float in a state of insensible equilibrium. See Atwood's excellent memoir on Floating Bodies.

17. If the centre of gravity of the solid be situated below the metacentre, the solid will float with stability; if the centre of gravity coincide with the metacentre, the stability vanishes, and the solid will float indifferently in all positions; if the centre of gravity be placed above the metacentre, the equilibrium of the body will be that of instability.

The metacentre is the point S, the intersection of the axis SL passing through G the centre of gravity round which the body revolves through an indefinitely little angle, with SF the line of support; now if G be situated below S, as represented in the figure, the line of support is moved towards those parts which are immersed by the inclination, and therefore the equilibrium is that of stability; if S coincide with G, the line of support coincides with KO, and therefore the body will float indifferently; if S fall below G, that is, if SQ pass between E and R, the line of support will be moved towards those parts which are elevated by the inclination, and therefore the position of the body will be instable.

18. The distance of the metacentre from the centre of gravity is $=$ the sum of all the $\frac{AB^3}{12V}$ $- GO$.

For

For $SG = \frac{GZ}{s}$; but $GZ =$ the sum of all the $\frac{AB^3 . s}{12 V}$ $- GO \times s$.

19. The measure of stability is the product of the weight of the body, the distance between the centre of gravity and metacentre, and the sine of angle through which the body is inclined.

For if W be the weight of the body, the measure of stability is $W \times GZ$; but $GZ = SG \times s$.

20. The oscillations of a floating body in finite angles are not analogous to the oscillations of a cycloidal pendulum.

Because the force of stability varies in a proportion very different from that of the distance from quiescence, unless the arcs of vibration be of evanescent magnitude.

LECTURE

LECTURE V.

1. THE ſpecific gravity of a lighter ſolid is to that of the fluid on which it floats, as the part immerſed to the whole body.

This method of determining the ratio of the ſpecific gravities of the ſolid and fluid is not very exact, becauſe it is difficult to determine the magnitude of the part immerſed, upon account of the agitation of the ſolid, and the attraction or repulſion which may ſubſiſt between it and the fluid; and even though the magnitude of the part immerſed were accurately known, yet ſtill its ratio to the whole could ſeldom be accurately aſcertained.

2. The ſpecific gravities of two ſolids both lighter than the fluid, are to each other directly as the magnitudes of the parts immerſed, and inverſely as the whole bodies.

3. If a body be lighter than the fluid its ſpecific gravity may be thus found: take another ſolid heavier than the fluid, ſo that both together may ſink; balance the heavier in water, and add the lighter to the ſcale to which the heavier is attached, the weight which reſtores the equilibrium is the weight of the lighter ſolid in air; then connect the lighter with the heavier under water, the weight which reſtores the equilibrium is the weight of a portion of the fluid of the ſame bulk with the lighter, therefore the ſpecific gravity of the ſolid is to that of the fluid, as the former weight to the latter.

If the ſolid, whoſe ſpecific gravity is ſought, be diſſolvable in water, let ſome other liquid be made uſe of, which will not diſſolve it; and the ſpecific gravity of the ſolid will be to that of water, in a ratio compounded of the ratios of its ſpecific gravity to that of the intermediate fluid, and of the intermediate fluid to water.

If the body to be examined conſiſt of ſmall fragments, they may be put into a ſmall bucket and weighed, and their ſpecific gravity determined as in this article, where a lighter body is connected with an heavier.

4. The ſpecific gravities of two fluids, both heavier than a ſolid, are to each other reciprocally as the parts of that ſame ſolid reſpectively immerſed in them.

5. The hydrometer is an inſtrument, which conſiſts

fiſts of an even ſlender ſtem graduated, and inſerted into a hollow ball, ſo as that it may ſink in various fluids to different depths marked on the ſtem; the ſpecific gravities of thoſe fluids will be inverſely as the parts of the hydrometer, which are immerſed in them.

If it be required, that different hydrometers ſhould be graduated in the ſame manner, and by the ſame numbers on the ſtem denote the ſame ſpecific gravity, let there be two ſtandard fluids fixed on, whoſe ſpecific gravities are known and conſtant; then ſink the hydrometer in the heavieſt ſtandard fluid, ſo as that it ſhall riſe above the ball, and note the place; then ſink it in the lighter, and obſerve where it cuts the ſtem; the ratio of the ſpecific gravities being given, the ratio of the parts immerſed is alſo given, and therefore the ratio of the whole to the ſtandard interval intercepted between the two points, where the two ſtandard fluids cut the ſtem; which ſtandard interval may then be divided in any propoſed manner. Or the graduation may be thus performed, more conveniently, with only one ſtandard fluid; immerſe the hydrometer in this ſtandard fluid, as ſuppoſe diſtilled water, and mark the point to which the fluid riſes on the ſtem, then add to the ſtem a weight which bears the ſame proportion in each to the weight of the whole hydrometer, and obſerve the point to which the inſtrument deſcends, this will be the other extremity of the ſcale, which is to be divided into the ſame number of parts. If the ſtem have four faces, it may be

made equivalent to one of four times the length by three different weights which may be fucceffively fixed on the top of the ftem.

There is another method of comparing the fpecific gravities of fluids by a feries of glafs balls, varying by fmall degrees in fpecific gravity; the degree of each is firft accurately determined; thefe balls being thrown into a fluid, fome will fink, others float, and one only will remain fufpended, which is of the fame fpecific gravity with the fluid; but this method is obvioufly inaccurate. There is another much more accurate, and fufficiently expeditious, which is by filling a given veffel with the different fluids fucceffively, and finding the weights of the contents: the ratio of the weights will be that of the fpecific gravities.

6. If a veffel contain two immifcible fluids, as water and mercury, and a folid of fome intermediate fpecific gravity be immerfed under the furface of the lighter fluid, and float on the heavier; the part of the folid immerfed in the heavier fluid is to the whole folid, as the difference between the fpecific gravities of the folid and lighter fluid to the difference between the fpecific gravities of the two fluids.

Let the fpecific gravity of the heavier fluid be a, the part of the folid immerfed in it A; the fpecific gravity of the lighter b, the part of the folid immerfed in it B; the folid itfelf $A+B$, and its fpecific gravity c; then we have $aA+$

bB

$bB = cA + cB$, and $aA - cA = cB - bB$; therefore $c-b$: $a-c$ :: A : B, and $c-b$: $a-b$ :: A : $A+B$.

Hence it appears, that the common rule for aſcertaining the ſpecific gravities of a fluid and a lighter ſolid, by the ratio of the part immerſed to the whole, is not accurately true; becauſe the air is a heavy fluid, and therefore every ſolid floating on a fluid is in fact a ſolid of intermediate ſpecific gravity floating between two immiſcible fluids. So that to render the rule accurate, we muſt ſubduct the number expreſſing the ſpecific gravity of air from the two numbers expreſſing the ſpecific gravities of the ſolid and the fluid on which it floats.

7. If two fluids of different ſpecific gravities be mixed together in any proportion, ſo as to compoſe another fluid of ſome intermediate ſpecific gravity, the magnitude of the heavier fluid will be to the whole, as the difference between the ſpecific gravities of the reſulting and lighter fluids to the difference between the ſpecific gravities of the two given fluids.

This ratio is true only on ſuppoſition that the magnitude of the two fluids when mixed is equal to the ſum of their magnitudes when ſeparate; but this is ſeldom the caſe, owing to the chemical action of the two fluids on each other. The rule therefore is chiefly of uſe in determining the quantity of penetration or rarefaction, by comparing the computed magnitudes or denſities, with thoſe which are diſcovered by obſervation.

8. If

8. If M be the magnitude, S the ſpecific gravity, and W the weight in avoirdupois ounces of any body, the ſpecific gravity of water being unity, then will $M = \frac{W}{1000\,S}$, eſtimated in cubic feet.

Becauſe a cubic foot of water weighs juſt 1000 ounces avoirdupois. But in philoſophical ſubjects, the weights of bodies being for the moſt part ſmall, are eſtimated in troy ounces or grains, the magnitudes being referred to a cubic inch as a ſtandard; and ſince a cubic inch of water weighs 253.18 grains, or $\frac{253.18}{480} = ,52746$ of an ounce, the magnitude of the ſolid eſtimated in cubic inches $= \frac{W}{253.18\,S}$, when the weight W is in grains; or $= \frac{W}{,52746\,S}$, when the weight is eſtimated in troy ounces.

The capacity of an irregular veſſel is determined in the ſame manner, by determining the weight, and thence the magnitude of the water which fills it.

9. The weight W of a body eſtimated in avoirdupois ounces $= M \times S \times 1000$, the magnitude being given in cubic feet.

The weight eſtimated in grains $= M \times S \times 253.18$, the magnitude M being given in cubic inches; and if eſtimated in troy ounces, $W = M \times S \times ,52746$.

AEROSTATICS.

AEROSTATICS.

LECTURE I.

1. AIR is a ponderous fluid.

2. If a glaſs tube more than thirty one inches in length, one extremity of which is cloſed, be filled with mercury, and the other extremity be immerſed in a veſſel of the ſame fluid; the mercury in the tube will deſcend from the upper extremity, and will remain ſuſpended at ſome altitude, between 28 and 31 inches from the ſurface of the external mercury.

This

This inſtrument is called a barometer; becauſe the weight of a column of mercury, whoſe baſe is the orifice of the tube, and altitude equal to that of the mercury in the tube above the external ſurface, is equal to the weight of a column of air extending to the top of the atmoſphere, and whoſe baſe is the ſame orifice. And ſince the weight of this column of quickſilver, cæteris paribus, is as its altitude, it follows, that the weight of the air is proportional to the altitude of the mercury in the barometer. This is the Torricellian experiment.

The altitude, at which the mercury is ſuſtained in the barometer above the ſurface of the external mercury is called the ſtandard altitude. The preſſure of the atmoſphere is equal to about fifteen pounds avoirdupois, at the medium height of the barometer or $29\frac{3}{4}$ inches, upon every ſquare inch: for a cubic foot of mercury is nearly $=$ 13600 ounces; therefore a cubic inch $= 8$ ounces nearly, and $29\frac{3}{4} \times 8 = 238$ ounces $= 15$ pounds nearly.

3. The perpendicular altitude of the mercury in any number of tubes thus immerſed, will be the ſame, whatever be their bores or poſition.

For the force preſſing the quickſilver upwards is the weight of a column of air reaching to the top of the atmoſphere, whoſe baſe is the orifice of the tube; the force ſuſtaining the quickſilver is therefore, in different tubes, always directly proportional to the orifice; the force of the quickſilver therefore preſſing downwards, being equal to the force preſſing it up, is as the orifice; but the force of

of the quickſilver is alſo as its perpendicular altitude × the orifice; therefore the perpendicular altitude × the orifice is : : the orifice; that is, the perpendicular altitude in all the tubes is the ſame.

4. If the tube be leſs than 28 inches long, the mercury will remain contiguous to the upper ſurface of the tube.

Becauſe the preſſure of the air, when leaſt, is equivalent to the weight of a column of mercury, whoſe altitude is twenty-eight inches.

5. If a barometer be placed under the receiver of an air pump, as the air is exhauſting, the mercury deſcends, until it is nearly on a level with the mercury in the reſervoir.

6. If a tube more than thirty-one inches long, open at both ends, communicate at one extremity with the reſervoir of an air-pump, and the other be immerſed in a veſſel of quickſilver expoſed to the external air, as the air is exhauſting, the mercury aſcends in the tube, until it nearly attains the ſtandard altitude.

A tube thus applied is called a gage. From the two preceding experiments it follows, that the ſuſpenſion of the mercury in the barometer is neither cauſed by the fuga vacui of the Peripatetics, nor the mercurial funicle of Linus, but ſolely by the weight of the atmoſphere.

7. If a barometer whoſe lower extremity is immerſed in a baſon of mercury, be ſuſpended from

the arm of a balance, the weight neceſſary to be put into the oppoſite ſcale, in order to preſerve the equilibrium, will be equal to the weight of the tube, and of as much mercury as ſtands in the tube above the level of that in the baſon.

It might at firſt be ſuppoſed, that the weight requiſite to balance the barometer would be equal only to the weight of the tube, ſince the mercury is ſuſtained by the preſſure of the atmoſphere. But it is to be conſidered, that the weight which balances the barometer is not actually the counterpoiſe of the mercury within the tube; but of an equivalent column of air which preſſes upon the crown of the tube; for the weight of this column of air is not counterbalanced, and ſo rendered inefficient by an equal preſſure below, as it is when the tube is empty, that counterbalance being now otherwiſe employed in bearing up the quickſilver within the tube.

8. If a tube hermetically ſealed, and more than thirty-four feet long be filled with water, and inverted into a veſſel of the ſame fluid, it will ſink down and ſubſide at the altitude of thirty-four feet above the ſurface of the external water.

This is called the Paſchalian experiment, from its maſter the celebrated Paſchal, who contrived it in order to refute the opinion of the Peripatetics, who held, that the upper part of the barometer deſerted by the mercury was abſolutely filled by extremely ſubtile ſpirits ſeparated from the

the quickfilver in the tube. For if this were fo, the more fpirituous the fluid, the lower would it fubfide in the tube, whereas wine, which is manifeftly more fpirituous than water, was neverthelefs found to be fuftained at a greater altitude.

LECTURE II.

1. If a ſlender horizontal tube be inſerted into a larger tube hermetically ſealed, whoſe altitude is greater than thirty-one inches, and the vertical tube be filled with mercury, any variation of the ſtandard altitude will be to the contemporaneous variation in the length of the horizontal cylinder, inverſely as the ſquares of the diameters.

This is called the rectangular barometer. The advantages ariſing from being able to obſerve very minute variations in the height of the mercury have given riſe to a great variety of barometers of different conſtructions, all however founded on the ſame principle.

2. If a vertical barometer be bent at the height of twenty-eight inches, ſo as that the perpendicular altitude of the ſealed extremity ſhall be more than

than thirty-one inches above the level of the mercury in the reſervoir, the variation of the height of the mercury in this barometer will be to the contemporaneous variation of the height in the perpendicular barometer, as radius to the ſine of the angle which the bent part makes with the horizon.

From the length of the ſcale in the rectangular barometer, the attrition of the mercury againſt the ſides of the tube, and the ſudden aſcents and deſcents of the quickſilver, the continuity of the fluid is apt to be deſtroyed. To remedy this defect the diagonal barometer was contrived, in which the ſcale of variation might be greater than in the upright barometer, though not ſo great as in the rectangular, and therefore the riſe and fall more regular; but the want of exactneſs in the termination of the ſurface of the mercury, by which its height is denoted, has been found more than ſufficient to counterbalance the advantage of an enlarged ſcale. The vertical barometer is therefore now generally uſed.

3. The cautions to be obſerved in the conſtruction of a barometer are 1ſt. That the ſurface of the mercury in the reſervoir be always kept at the ſame altitude. 2dly. If it be not, the ciſtern ought to be cylindrical, and the ratio of its diameter to that of the tube accurately aſcertained, and the ſcale divided accordingly. 3dly. A Nonius or Vernier

nier ſhould be applied to the ſcale of variation. 4th. The barometer tube ſhould be truly cylindrical. 5th. The inner ſurface of the tube ſhould be perfectly clean. 6th. The mercury ſhould be purged from all heterogeneous matter. 7th. It ſhould be carefully freed from all aerial particles; which is generally done by boiling the mercury in the tube.

4. If a line conſiſt of equal parts, each equal to p, and it be required to ſubdivide each of theſe diviſions into n equal parts, take another line, called the nonius, equal to $n+1$ parts, each of which is equal to p, and divide this nonius into n equal parts: now if the nonius be applied to the given line, ſo that its extremity ſhall coincide with any diviſion of the ſcale, the diſtance between the two firſt diviſions from coincidence will be $\frac{p}{n}$; the diſtance between the two next will be $\frac{2p}{n}$; the diſtance between the two next, will be $\frac{3p}{n}$; and ſo on, until the diſtance between the two laſt diviſions becomes $\frac{np}{n} = p$.

Since the nonius and given line, whoſe lengths are $n+1$ and n parts, each of which is equal to p, are divided into the ſame number of parts n, each part of the nonius will be to each part of the ſcale directly as the magnitude of theſe

these lines, that is, as $n+1$ to n, or as $1+\frac{1}{n}$ to 1; that is, each part of the nonius exceeds each part of the scale by $\frac{1}{n}^{th}$ part of p, one of the divisions of the scale. Therefore if the nonius be applied to the given line, so as that its extremity shall coincide with any division of the scale, the distance between the two first divisions from coincidence will be $\frac{p}{n}$; the distance between the two next will be $\frac{2p}{n}$, and so on.

It is evident, that the same subdivision may be effected by making the nonius $= \overline{n-1}$ parts of the scale, and dividing it into n parts as before.

When the divisions of the nonius are $\overline{1-\frac{1}{n}}|^{th}$ part of a division of the scale, they will each be less than this division by $\frac{1}{n}^{th}$ part; but as larger divisions are more accurate than smaller, it seems better to make the nonius $= \overline{n+1}$ parts of the scale.

The numbers on the nonius and scale will observe a contrary or the same order, according as the divisions of the nonius are $\overline{1+\frac{1}{n}}|^{th}$ part, or $\overline{1-\frac{1}{n}}|^{th}$ part of the divisions of the scale.

If the index of the nonius cut off a part which is not an aliquot part of one of the divisions of the limb, it is evident

evident that no two divisions of the nonius and limb will coincide. To remedy any sensible error arising from this, a micrometer is sometimes applied.

5. If mercury be poured into a conical tube, the smaller end of which is hermetically sealed, and the tube be then inverted, the mercury will be sustained at the standard altitude.

When the pressure of the atmosphere is increased, the mercury is forced up into a higher part of the tube, because the quantity of mercury being given, it will occupy a greater length of the tube where it is narrower; the contrary will happen when the pressure of the air is diminished. This is called the conical barometer.

6. The variation in the scale of the conical barometer is to the contemporaneous variation in the scale of the vertical barometer, as the square of the diameter of the lower surface of the mercury to the difference of the squares of the diameters of the lower and higher surfaces.

7. The air is an elastic fluid.

The elasticity of air differs from the elasticity of solid bodies in this, that when the compressing force is removed, the air occupies a much greater space than it did before; whereas solid elastic bodies only resume their former magnitude and figure.

8. Heat increases the elasticity of air, and cold diminishes it.

It has been found by experiment, that the elasticity of the air is increased one 435th part, by each degree of heat expressed by Farenheit's thermometer, of which there are 180 between the boiling heat and the freezing of water.

9. The air is also a compressible fluid.

Air can undergo the strongest compression without decomposition; Hales by compression rendered air almost twice denser than water, nevertheless it appeared to suffer no change.

10. The elastic force of the air is equivalent to the force which compresses it.

From this property it is, that the small quantity of air contained in animal and vegetable bodies is able to counteract the violent pressure of the atmosphere, which would otherwise destroy the arrangement of their parts.

11. The spaces into which a given quantity of air is compressed, are reciprocally as the compressing forces.

12. The density of the air is directly as the compressing force.

We should observe, that the rule thus laid down must be understood with some limitation, for experience proves it defective after a certain degree of condensation has been effected: thus after air has been compressed into a fourth part of its volume, the force requisite to produce a certain degree of farther condensation is greater than that indicated by the rule.

13. The density of the air is different at diffe-

rent altitudes above the earth's ſurface, the inferior ſtrata being more compreſſed.

14. The particles of air repel each other with forces which vary inverſely as the diſtances between their centres.

15. If a tube of a ſmall bore be inſerted into a glaſs ball, and a ſmall quantity of mercury put into the tube, the aſcent and deſcent of the mercury in the tube will depend on the denſity of the air.

This inſtrument is called a manometer; it differs from the barometer in this, that the one meaſures the weight, the other the denſity of the air, which depends not only on the weight of the atmoſphere, but alſo on the action of heat and cold.

If a ſcale be placed on the ſide of the tube, marking the degrees of dilatation ariſing from an increaſe of heat, in the mean height of the barometer or 29.7 inches; the manometer will, in this ſtate, ſhew the degrees of heat in the ſame manner as a thermometer. But if the air become lighter or heavier, that which is incloſed in the ball, being leſs or more compreſſed, will either occupy a proportionally greater or leſs ſpace, and the inſtrument will ſhew the differences in the denſity of the air ariſing from the changes in its weight and heat conjointly.

LECTURE

LECTURE III.

1. THE altitude of the mercury in a barometer elevated above the earth's ſurface is obſerved to be leſs than the ſtandard altitude at the earth's ſurface.

This diminution is the height of a column of mercury, whoſe weight is equal to that of the intermediate column of air between the earth's ſurface and the elevated barometer.

If a barometer be carried to an altitude of 54 feet, the mercury is obſerved to ſink about $\frac{1}{20}$ of an inch.

2. The height of an homogeneous atmoſphere is to the height of the mercury in the barometer, as the ſpecific gravity of the mercury to that of the air.

Since the ſpecific gravity of air is to that of water, as 1,2318 to 1000; and water to mercury as 1 to 13.6; and

ſince the ſtandard altitude is thirty inches or 2½ feet, the height of the homogeneous atmoſphere above the ſurface of the earth is 27600 feet.

If the ſpecific gravity of the quickſilver be variable, its mean height being ſtill ſuppoſed 2.5 feet, the height of the homogeneous atmoſphere will vary in the ſame proportion. Now the ſpecific gravity of every material ſubſtance varies with its temperature; and it is found by experiment, that when the mercury in the barometer ſtands at its mean height of thirty inches, the difference between the lengths of that column of mercury ſucceſſively heated to the temperature of boiling water, and cooled to that of melting ice, is $\frac{1}{53,333}$ of the mean height. But the whole extent of Farenheit's ſcale between theſe two degrees of temperature is 180°; therefore the change of the height of the mercury in the barometer, due to one degree of the thermometer, is always $= \frac{1}{53,333} \times \frac{1}{180} = \frac{1}{9600}$ part of the whole height. In the ſame manner, the expanſion of air is equal to the $\frac{1}{435}$ part of its bulk, for each degree of Farenheit.

3. The height of the atmoſphere, which is uniformly of the ſame denſity with the air at any altitude above the earth's ſurface, is a conſtant quantity.

For let the ſpecific gravity of air be to that of mercury as n to 1, and let a be the altitude of the mercury in the baro-

barometer at any elevation, and b the height of the homogeneous atmosphere sought, then is $b = \frac{a}{n}$; but n will be always a measure of the air's density and pressure, and therefore is always directly as a, consequently $\frac{a}{n}$ is a constant quantity.

4. The height of the homogeneous atmosphere, at any given station, is not varied by any difference in the weight of the air.

This follows from the last article, the effects of heat and cold not being considered. But if, the pressure remaining the same, the elasticity be varied, the density, and consequently the height of the homogeneous atmosphere will be varied in the same proportion.

5. If a right line perpendicular to the surface of the earth be made the axis of a curve whose ordinates, drawn at right angles to different points of the axis, express the densities of the air in those points, this curve will be the logarithmic.

Hence if any number of distances from the surface of the earth be taken in arithmetical progression, the densities of the air at those distances will be in geometrical progression.

6. The subtangent of this logarithmic curve is equal to the height of the homogeneous atmosphere.

7. The difference of elevations of any two places above

above the ſurface of the earth is = height of the homogeneous atmoſphere multiplied by the log. of the ratio of the heights of the mercury at thoſe ſtations, and applied to the modulus of the ſyſtem.

This is true on the following hypotheſes, 1ſt. That the force of gravity is the ſame at all altitudes. 2dly. That the ſpecific gravity of the mercury is conſtant. 3dly. That the height of the homogeneous atmoſphere is invariable.

8. If gravity be ſuppoſed to vary in the inverſe duplicate ratio of the diſtances from the earth's centre, and l be the logarithm of the ratio of the heights of the mercury in the barometer at two different ſtations, h the height of the homogeneous atmoſphere, s the ſemidiameter of the earth, and a the difference of the elevations, then will $a = \frac{hl}{m} \times \frac{s+a}{s}$,

Since a, the greateſt height which is acceſſible by man, is not four miles, and therefore is evaneſcent with reſpect to s, $s+a$ may be conſidered as equal to s, and therefore $a = \frac{hl}{m}$, as before,

9. If M, m repreſent the heights of the mercury in the lower and higher ſtations, then $a = \frac{27600}{,43429448} \times L. \frac{M}{m} = 63551 \times L. \frac{M}{m}$, in feet, $= 10592 \times L.$

$\frac{M}{m}$, in fathoms, the air being in its mean temperature of 55°.

10. If M and m exprefs the heights of the quickfilver in the lower and higher ftations, and n be the difference between the temperature in thofe ftations, in degrees of Farenheit, the logarithm of the ratio of the heights, in the fame temperature, will be $= \text{Log.}\frac{M}{m} \pm n \text{ Log. } \frac{9600}{9601}$, according as the temperature in the upper ftation is lefs or greater than that in the lower ftation.

11. If Farenheit's thermometer ftand at 55°, the length of the atmofpherical fubtangent requires no correction; but there is an error in excefs or defect of $\frac{1}{435}$ part of the height, for every degree of Farenheit above or below 55°.

12. The formula in Art. 9. is rendered more convenient for ufe, by making $a = 10000 \times L.$ $\frac{M}{m}$, and fuppofing the temperature of the air to be 31°.

For as 10592 : 10592 — 10000 : : 18 : 1 : : 435 : 24, the number of degrees of difference in temperature correfponding to the given change in the altitude; but 55° — 24° = 31°. See Hutton's Math. Dict. Art. Barometer.

13. The precepts for the meafuring of altitudes by the barometer are as follows:

1 ft. Obferve

1 st. Obſerve the heights of the barometer at the bottom of the height to be meaſured; together with the temperature of the mercury, by a thermometer attached to the barometer; and alſo the temperature of the air in the ſhade, by another thermometer, which is detached from the barometer.

2ndly. Let the ſame thing be done at the ſummit of the ſaid height.

3dly. Let the altitudes of the mercury be corrected, by reducing it to the ſame temperature; viz. by augmenting the height of the mercury in the colder temperature, or diminiſhing that in the warmer by its 9600th. part, for every degree of difference between the two; and the altitudes of mercury ſo corrected are what are denoted by M and m, in the above formula, Art. 12.

4thly. Take out the common logarithms of the two corrected heights of the mercury, and ſubtract the leſs from the greater, then cut off from the right hand ſide of the reſulting logarithm, ſo many places for the decimals as the number of places of the logarithms exceeds four: ſo ſhall thoſe on the left be fathoms in whole numbers, by Art. 9.

5thly. Correct the number laſt found, for the difference of the temperature of the air thus: take half

half the ſum of the two temperatures of the air, ſhewn by the detached thermometers, for the mean temperature; and for every degree which this differs from the ſtandard temperature of 31° take ſo many times the 435th. part of the fathoms above found; and add it, if the mean temperature be more than 31°, but ſubtract it, if the mean temperature be leſs; ſo ſhall the ſum or difference be the true altitude in fathoms; or being multiplied by ſix, it will give the altitude in Engliſh feet. See Phil. Trans. for the year 1774, and Hutton's Math. Dict. Art. Barometer.

LECTURE IV.

1. IF the capacity of the barrel of an air-pump : the capacity of the receiver, as $b : r$; after every ſtroke of the pump, the quantity of air extracted : the quantity before the ſtroke, as $b : b+r$.

Hence the quantities of air extracted at any number of ſucceſſive ſtrokes of the pump, form a geometrical ſeries.

2. The quantity of air remaining after every ſtroke of the pump : quantity before the ſtroke, as $r : b+r$.

Hence the quantities which remain, form a decreaſing geometrical ſeries; conſequently the whole primitive quantity of air can never be extracted.

3. After every ſtroke of the pump, the denſity of the air is diminiſhed in the ratio of $b+r$ to r; and

and hence after s number of ſtrokes, it is diminiſhed in the ratio of $\overline{b+r}|^{s} : r^{s}$.

4. The defects of the mercury in the barometer gage from the ſtandard altitude, after any number of ſucceſſive ſtrokes, form a geometrical ſeries, whoſe terms are in the ratio of $b+r : r$.

5. The altitudes of the mercury in the gage, at the ſame time, form a geometrical ſeries, the ratio of whoſe terms is $b+r : b$.

6. When the air is rarefied n times, the number of ſtrokes $= \dfrac{\log. n}{\log. \overline{b+r} - \log. r}$.

7. After every deſcent of the piſton in the condenſer, one barrel of air, in its natural ſtate, is forced into the receiver.

8. If the capacity of the receiver : the capacity of the barrel, as $r : b$, then after s deſcents of the piſton, the denſity is increaſed in the ratio of $r : r+bs$.

9. After any number of deſcents, the denſity is increaſed in arithmetical progreſſion.

10. If the gage be an horizontal tube, cloſed at one end, the ſpaces which the air occupies after any number of ſucceſſive deſcents of the piſton, will decreaſe in muſical progreſſion.

11. If the gage be an open vertical tube, communicating with the external air, the altitude of

 the

the mercury in the gage after any number of ſtrokes of the pump, will increaſe in arithmetical progreſſion.

12. The ſpecific gravity of air may be accurately aſcertained by means of the air-pump.

To the neck of a glaſs bottle, made in the form of a Florence flaſk, adapt a cap and valve opening outwards, ſcrew it on the pump, and exhauſt it to a known degree; then from the weight of the bottle before and after exhauſtion, you have the weight of the exhauſted air; and from the ratio of the height of the quickſilver in the gage to the ſtandard altitude, you know the proportion which the exhauſted part bears to the whole air originally in the veſſel, whoſe weight is therefore known. Subtract this weight from the weight of the veſſel when full of air, there remains the weight of the veſſel itſelf; fill it with water and weigh it, and from this weight ſubduct the weight of the veſſel; the remainder is the weight of a bulk of water of the ſame magnitude with the air which fills the veſſel, and whoſe weight was alſo previouſly aſcertained.

By a mean of ſeveral experiments it appears, that the ſpecific gravity of air is to that of water, as 1.232 to 1000, when the barometer ſtands at thirty inches, and in the mean temperature of 55° of Farenheit's thermometer.

LECTURE

HYDRAULICS.

LECTURE I.

1. HYDRAULICS is that part of Natural Philosophy which treats of the motion of coherent fluids.

2. There are four causes of the motion of such fluids, the gravity or pressure of the fluid, the weight of the air, the elastic force of the air, and the elastic force of steam.

3. The velocity of water in any pipe, kept constantly full, is inversely as the different sections of the pipe, perpendicular to the axis.

4. If

4. If water be poured into a cylindrical tube, which is turned up at the ends, the water will always rise to the same level in the upright parts.

This is called the water level.

5. The difference between the true and apparent level of any two places is equal to the square of the horizontal distance between the places divided by the diameter of the earth.

The true level is about an inch below the apparent in 1930 feet; the general allowance in water courses is $4\frac{1}{2}$ inches in a mile; in narrow tubes however 5 feet fall will be requisite in a mile.

6. If water flowing through a small orifice in the bottom of a vessel be kept constantly at the same height in the vessel, by being supplied as fast above as it runs out below, the velocity of the effluent water will be equal to that which a heavy body would acquire in falling down the height of the fluid above the orifice.

Let h be the height of the water in the vessel, a the area of the indefinitely little orifice, t the thickness of the lowest plate of water; the force which impels the lowest plate, is the weight of a column of water equal to ha; and since the area of the lowest plate of water is indefinitely less than the base of the vessel, and the water is supposed to continue at the same height, the quantity of matter moved, during the discharge of the plate, is at; therefore the accelerating

lerating force $=\frac{ha}{at}=\frac{h}{t}$; and the ſpace, through which the matter moved is accelerated, is t, therefore the velocity $=\sqrt{4gSF}, = \sqrt{4\,gh}$, equal to the velocity which a heavy body would acquire in falling through the height h.

7. The velocity of the effluent water is proportional to the ſquare root of the altitude of the water above the orifice.

8. The force with which the effluent water impinges againſt any quieſcent body, is proportional to the altitude of the fluid above the orifice.

For the force is as velocity $\times$ quantity of matter; but the quantity diſcharged in a given time is as the velocity; therefore the force is as the ſquare of the velocity, that is, as the height of the fluid.

9. When a veſſel is left gradually to diſcharge itſelf by an orifice in the bottom, the velocity of the efflux will be uniformly retarded, according to the law of a heavy body projected perpendicularly upwards.

10. The quantities of water in a priſmatical or cylindrical veſſel diſcharging itſelf through an aperture in the bottom, decreaſe in equal times, as the ſeries of odd numbers 1, 3, 5, 7, &c. taken in an inverted order.

11. The quantity of water contained in a cylindric veſſel is half that which would be diſcharged in

in the time of the entire gradual evacuation of the veſſel, if the water be kept always at the ſame altitude.

12. The time in which a veſſel would evacuate itſelf through an aperture in the bottom, is equal to the time in which a body would fall through the height of the fluid $\times \frac{\text{baſe}}{\text{orifice}}$.

This theory is true only on the hypotheſis, that the water flows out, without reſiſtance, in a cylindrical or priſmatic form, correſponding to the aperture through which it flows; but in fact, all the particles preſſing towards the orifice, they iſſue through it in a converging direction, that is, in a conical not cylindrical form, by which means the quantity diſcharged is diminiſhed; and this diminution is found to be nearly in the ſubduplicate ratio of 2 to 1.

13. The times in which two cylindric or priſmatic veſſels will ſeverally empty themſelves, are to each other in a ratio compounded of the direct ſimple ratio of their baſes, and ſubduplicate of the altitudes, and the inverſe ratio of the apertures.

14. If on the altitude of the fluid as a diameter we deſcribe a ſemicircle, the horizontal ſpace deſcribed by the fluid ſpouting from any point in the diameter, will be as the ordinate drawn from that point.

15. If the water ſpout horizontally through a pipe,

pipe, the diſtance to which it ſpouts is leſs than if it iſſued through an adjutage inſerted in the hole.

Becauſe the friction in the latter caſe is leſs.

16. On the contrary if the pipe be inſerted perpendicularly into the bottom of a veſſel, the water will diſcharge itſelf ſooner through the pipe, than through a ſimple orifice, and ſtill quicker as the pipe is longer.

This is cauſed by the preſſure of the atmoſphere, for in vacuo the time of the efflux is not varied by the inſertion of the pipe.

17. The quantity of water diſcharged through a a perpendicular pipe in form of a truncated cone, the narrower part being fixed in the orifice, is greater than through a cylindrical pipe of the ſame length, its diameter being equal to that of the orifice.

Becauſe the inſertion of the pipe increaſes the velocity of the diſcharge, and therefore alſo the reſiſtance of the air: therefore the diameter of the vein has a tendency to enlarge itſelf, which the widening cone admits. But the cylindrical pipe, not admitting of this augmentation of diameter, obſtructs the efflux.

18. If the water iſſuing from the orifice in a veſſel be directed upwards, it will be projected nearly to the height of the water above the orifice.

LECTURE II.

1. THE velocity with which water ſpouts into a vacuum, from an orifice in a veſſel, by the preſſure of the atmoſphere alone, is equal to the velocity, which a heavy body would acquire in falling down the height of thirty four feet.

For the moving force in this caſe is the preſſure of the atmoſphere, which is equivalent to a column of water of thirty four feet, and therefore the velocity will be the ſame as if an aperture were made in the bottom of a veſſel thirty four feet high, and filled with water; that is, will be equal to the velocity which a heavy body would acquire in falling through thirty four feet.

Though the preſſure of the atmoſphere be equal to about $14\frac{1}{4}$ pounds upon every ſquare inch, yet on account of the friction of machines and other impediments, the efficient preſſure

preſſure, forcing the water into a vacuum, is not above ſe- ven or eight pounds on every ſquare inch.

2 If the legs of a bent tube filled with water be immerſed in two veſſels of water of unequal altitudes, the preſſure of the air will drive the water from the higher to the lower veſſel, till it ſtands in both at the ſame level.

This is the common ſiphon. The force generating the flux is the difference of the weights of equal columns of air and water, whoſe baſe is the ſection of the pipe, and altitude the difference of the heights of the water in the veſſels.

3. Intermitting ſprings, or ſuch as flow and ſtop alternately, are natural ſiphons.

4. Reciprocating ſprings, whoſe waters flow and ebb by alternate reciprocations, are accounted for by their connection with intermitting ſprings.

5. The velocity of ſpouting water generated by the action of condenſed air, is in the ſubduplicate ratio of the condenſation, or of the elaſtic force of the air.

6. Steam being an elaſtic fluid, whoſe elaſticity is proportional to its denſity when the heat is the ſame, or proportional to the heat when the denſity is the ſame, may be applied in the ſame manner as air to the moving of water.

The ſteam raiſed with the ordinary heat of boiling wa-

ter, is almoſt 3000 times rarer than water, or about $3\frac{1}{2}$ times rarer than air, and its elaſticity is nearly equal to that of common atmoſpheric air; and it has been found, that, by great heat, ſteam may be rendered about five times ſtronger.

LECTURE

LECTURE III.

1. IF a ſtream of water impinge on the floats of an underſhot wheel, and eſcape from it, the very inſtant after it has made its impact, the quantity of water which actually impinges againſt the wheel, will be to the whole quantity which paſſes by it, in a given time, as the difference between the velocities of the water and the wheel to the abſolute velocity of the water.

Fig. 37. Let ST be the ſtream of water, and the float F firſt receive the impact from the water at F, and leave it at L; alſo let WF be to FL, as the abſolute velocity of the water to the velocity of the float. When F arrives at L, W will be at F, and taking $WV = FL$, all the water in the ſpace WV will paſs by the wheel, without impinging againſt it; for it cannot impinge on the float F, becauſe that float emerges from the water at L; neither can

can it impinge on the fubfequent float, becaufe it has already paffed it. Therefore the whole quantity of water which paffes by the wheel in a given time, is to that which actually impinges againft it, as *WF* to *VF*.

Hence we may correct the miftake of Mr. Waring, in his New Doctrine of Mills, who lays it down as a fundamental principle, that while the ftream is invariable, whatever be the velocity of the wheel, the fame number of particles or quantity of the fluid muft ftrike it fome where or other in a given time. See the 3d. vol. of the Tranf. of the American Society, page 144. or Hutton's Math. Dict. article Mill.

2. The force of the impinging water is as the fquare of the difference between the velocities of the wheel and water.

3. If w be a weight faftened to a line which is wound round the horizontal axis of an under-fhot water wheel, A the weight of a column of water equal to the force of the impact of the water on the wheel, when the wheel is quiefcent, v the velocity with which the water impinges on the float boards, y the velocity of the circumference of the wheel, a the radius of the wheel, and b the radius of the axle, then will the velocity of the wheel $y = v - v \times \sqrt{\frac{wb}{Aa}}$.

The difference of the velocities of the water and the wheel $= v - y =$ the abfolute velocity with which the water

water ſtrikes the wheel; and ſince the force of the ſtroke is as the ſquare of the velocity, we have $v^2 : \overline{v-y}|^2 :: A : f =$ the force of the water to turn the wheel, when its velocity is y; and $y = v - v \times \sqrt{\frac{f}{A}}$. Now the acceleration of the wheel will ceaſe, when the force of the water, to turn the wheel, is equal to the force of the weight which oppoſes it, that is, when $f \times a = w \times b$, or $f = \frac{wb}{a}$; therefore, for f ſubſtituting its value, the acceleration of the wheel will ceaſe when $y = v - v \times \sqrt{\frac{wb}{Aa}}$.

See Atwood's Rect. Motion, page 275. We are to obſerve however, that this concluſion is true only on the hypotheſis, that the water eſcapes from the wheel, as faſt as it impinges.

4. If the weight w vary, its moment will be the greateſt poſſible, when the wheel has acquired its uniform velocity, if the weight be $= \frac{4Aa}{9b}$.

Let the weight ſought be x, and ſince the uniform velocity of the wheel $= v - v\sqrt{\frac{xb}{Aa}}$, the uniform velocity of $x = \frac{vb}{a} - v\sqrt{\frac{xb^3}{Aa^3}}$; and the moment of $x = \frac{vbx}{a} - v\sqrt{\frac{x^3b^3}{Aa^3}}$; the fluxion of which being made $= 0$, we have $\dot{x} - \frac{3}{2}\dot{x}\sqrt{\frac{xv}{Aa}} = 0$, and $x = \frac{4Aa}{9b}$.

5. When

5. When the moment of the aſcending weight attached to the axle is a maximum, that weight will be $\frac{4}{9}$ of the weight, which would, if ſuſpended from the axle, balance the force of the ſtream.

For if $\mathfrak{Q}$ be ſuch a weight, ſuſpended from the axle, as would balance the ſtream acting at the circumference, then $\mathfrak{Q} = \frac{aA}{b}$; but the moment of x is a maximum when $x = \frac{4Aa}{9b} = \frac{4}{9}\,\mathfrak{Q}$.

6. The velocity of the circumference of the wheel will be $\frac{1}{3}$ of the abſolute velocity of the ſtream, when the moment generated in the weight, aſcending uniformly, is the greateſt poſſible.

For the uniform velocity of the wheel $= v - v\sqrt{\frac{wb}{Aa}}$; but when the moment of w is a maximum, $w = \frac{4Aa}{9b}$; therefore the velocity of the wheel, when the moment of the weight is the greateſt poſſible, is $= v - v\sqrt{\frac{4}{9}} = \frac{1}{3}v$.

7. If w be a weight to be raiſed by the action of a ſtream of water, whoſe force againſt the wheel is A, the radius of the wheel will be $= \frac{9wb}{4A}$, when the uniform

uniform velocity of the aſcending weight, attached to the axle, is a maximum.

For let the ſought radius of the wheel be x, that of the axle being b; then will the uniform velocity of the aſcending weight $= \frac{bv}{x} - v\sqrt{\frac{wb^3}{Ax^3}}$; of which if the fluxion be made $= 0$, we ſhall have $x = \frac{9wb}{4A}$.

8. The greateſt moment generated in the aſcending weight will be expreſſed by the quantity $\frac{4}{27}Av$.

For ſince the uniform velocity of the aſcending weight is $\frac{bv}{3a}$; and the weight moved $w = \frac{4\ Aa}{9b}$, the moment generated in the aſcending weight will be $\frac{4Av}{27}$.

9. If the velocity of the ſtream be given, the effect will be as the quantity of water expended.

For the effect is as $\frac{4}{27}\ Av$, that is, ſince $\frac{4}{27}$ is conſtant, in a ratio compounded of the force of impact and of the velocity of the ſtream; but the force of the impact is as the quantity of water expended $\times$ velocity; therefore the effect is as the quantity of water expended $\times$ the ſquare of the velocity; and therefore, if the velocity be given, the effect will be as the quantity of water expended.

10. The expence of water being the ſame, the effect will be as the ſquare of the velocity.

 11. The

11. The expence of water being the fame, the effect will be as the height of the head of water.

For the velocity is as the fquare root of that height, and therefore that height is as the fquare of the velocity.

12. The aperture being the fame, the effect will be as the cube of the velocity of the water.

For the effect is as the expence of water $\times$ velocity [2]; but the expence of water, if the aperture be given, is as the velocity.

13. If all the water which paffes by an under-fhot wheel be fuppofed to impinge againft it, the force of the ftream will be in the fimple direct proportion of the relative velocity.

Becaufe the number of particles which ftrike the wheel in a given time is given, whatever be the velocity of the wheel.

14. On the fame hypothefis, y, the velocity of the wheel, will be equal to $v - v \times \frac{wb}{Aa}$.

See Art. 3.

15. On the fame hypothefis, if the weight w vary, its moment, when a maximum, will be $= \frac{Aa.}{2b}$

Let the weight be x, fince the uniform velocity of the wheel is $v - \frac{vbx}{Aa}$, the uniform velocity of the afcending

weight

weight will be $\frac{vb}{a} - \frac{vb^2x}{Aa^2}$, and its moment $= \frac{vbx}{a} - \frac{vb^2x^2}{Aa}$; whose fluxion being made equal to o, we have $\dot{x} - \frac{2x\dot{x}b}{Aa} =$ o, and $x = \frac{Aa}{2b}$.

16. On the ſame hypotheſis, the velocity of the wheel will be equal to half the velocity of the ſtream, when the effect is a maximum.

For the velocity of the wheel $= v - \frac{vwb}{Aa}$; but when the effect is a maximum, $w = \frac{Aa}{2b}$; therefore the velocity of the wheel $= v - \frac{1}{2}v = \frac{1}{2}v$.

17. On the ſame hypotheſis, the greateſt moment generated in the aſcending weight will be $= \frac{1}{4}Av$.

For the uniform velocity of the aſcending weight $= \frac{bv}{2a}$; and the weight moved $w = \frac{Aa}{2b}$, therefore the moment $= \frac{1}{4}Av$.

18. In practice, the velocity of the wheel, when the machine is in its ſtate of greateſt perfection, will be between one third and one half of the velocity of the ſtream.

Becauſe the water does not all eſcape the inſtant after it has made its impact, but is confined by the channel for ſome time; ſo that the ſucceeding water which would

otherwiſe paſs by the wheel inefficaciouſly, drives the confined water againſt the floats; and therefore acts in the ſame manner, as if it actually impinged againſt the wheel. It has been found by experience, that when the moſt work is done in a given time, the velocity of the wheel lies much nearer to one half of the velocity of the ſtream than to one third. See Smeaton's Enquiry, page 77.

19. In an overſhot wheel, the machine will be in its greateſt perfection, when the diameter of the wheel is two thirds of the height of the water above the loweſt point of the wheel.

Fig. 38. If the wheel *SRVP* revolve with a velocity which a body would acquire in falling through the altitude $x = NS$, the height of the water above the wheel, the water will fall into the buckets without any impulſe, and produce its effect by its weight only. Let $A = NV$, the height of the water above the loweſt point of the wheel; the force of every portion of water E, in the buckets, to turn the wheel, will be equal to its weight multiplied into its perpendicular diſtance from the axis of motion $= E \times EF$; and the ſum of all the forces, in all the buckets, in the ſemicircle *SRV*, will be equal to the ſum of all the $E \times EF =$ the ſemicircle $SRV \times GL$, G being the centre of gravity of the ſemicircle, $=$ the quantity of water $SRV \times \frac{GL}{LR}$, acting at the extremity of the radius LR. But $GL = \frac{LR^2}{SR}$; therefore the force of all the water to turn the

wheel,

wheel, is = the quantity of water $SRV \times \frac{LR}{SR}$, acting at R, = the quantity of water SV acting at R. Let $LR = r$; then the whole force of the water $= r \times SV = r \times \overline{A - x}$. Now the velocity of the wheel is $= 2\sqrt{gx}$; therefore the force of the water to produce the circular motion of the wheel is $= r \times \overline{A - x} \times 2\sqrt{gx}$. If this be made a maximum, we shall have $x = \frac{1}{3}A$. Hence the height of the head of water above the wheel should be equal to one third of the whole height; and the diameter of the wheel equal to two thirds of the altitude, in order that the machine may produce the greatest effect.

20. The velocity of an overshot wheel, when the water produces its effect by its weight only, and the machine is in its state of greatest perfection, is to the velocity of an undershot wheel as $\sqrt{3}$ to 1, on hypothesis that all the water escapes from the undershot wheel, the moment after it makes its impact.

For A being the height of the water, the velocity of the overshot wheel will be $= \sqrt{\frac{4}{3}gA}$; and the velocity of the undershot $= \frac{1}{3}\sqrt{4gA} = \sqrt{\frac{4}{9}gA}$; therefore the the former is to the latter, as $\sqrt{\frac{4}{3}} : \sqrt{\frac{4}{9}} :: \sqrt{3} : 1$.

21. The efficacy of an overshot wheel is to that of an undershot wheel, the height of the water, aperture, and diameter being given, as 26 to 10 nearly.

If

If A be the height of the column of water, whofe weight is equal to the force of impact on the underfhot wheel, when quiefcent, fince the velocity of the wheel is equal to $\frac{1}{3}$ of the velocity of the ftream we fhall have this proportion, as $3^2 : \overline{3-1}|^2 :: A$: the weight which is equivalent to the force of the ftream, when the machine is in its greateft perfection; therefore if V be the velocity acquired in falling down the altitude A, the moment of the wheel will be $= \frac{4}{9} A \times \frac{1}{3} V = \frac{4}{27} AV$, when in its greateft perfection. In the overfhot wheel, the weight of the water acting at the circumference, is equivalent to $\frac{2}{3} A$; and the velocity when the machine is in its greateft perfection is $= \sqrt{\frac{4}{3} gA} = \sqrt{\frac{1}{3}} \times V$; therefore the moment of the overfhot wheel $= \frac{2}{3} A \times \sqrt{\frac{1}{3}} V$; and confequently is to that of the underfhot wheel, as $\frac{2}{3} \times \sqrt{\frac{1}{3}} : \frac{4}{27} :: 2,598 : 1$.

Mr. Ferguson obferves, that when there is but a fmall quantity of water, and a fall great enough for the wheel to lie under it, the overfhot wheel is the beft: but when there is a large body of water, and but little fall, the underfhot wheel is to be preferred.

22. There is no limit in theory to the weight which a given ftream of water can raife by means of a water wheel.

Becaufe either the radius of the wheel may be increafed, or that of the axle diminifhed without limit.

23. If there were no friction or other impediment, machines moved by the impact of water would

would not acquire their uniform velocity in any finite time whatever.

Fig. 31. For the abfolute force of the water on the wheel $= A \times \frac{\overline{v-y}^2}{v^2} = A + A \times \frac{y^2 - 2vy}{v^2}$; from which is to be fubducted the conftant retarding force of the weight fufpended from the axle, which let us fuppofe $= d$; then the abfolute accelerating force, upon the whole, is $A - d + A \times \frac{y^2 - 2vy}{v^2}$. Let the conftant part of the force $A - d$ be reprefented by the given line AC; and let the variable part $A \times \frac{y^2 - 2vy}{v^2}$ be equal to AK; confequently the abfolute accelerating force, upon the whole, is equal to KC; alfo let $AP = v - y$, be the relative velocity of the wheel; then KL, the moment of AK, will be directly as $2\dot{y} \times \overline{y-v}$, that is, as $2\,PQ \times AP$, that is, as $KC \times AP$, becaufe PQ, the moment of the relative velocity, is directly as KC, the abfolute accelerating force; therefore $KN \times KL$ is as $KL \times KC \times AP$, that is, as AP, becaufe $KL \times KC$ is conftant. Therefore the indefinitely little hyperbolic area $KNOL$ is as AP; and the hyperbolic area $ABOL$ is compofed of the particles $KNOL$ always proportional to the fpace defcribed with that velocity, the particle of time, in which KL is generated, being given. Confequently, when KC the abfolute accelerating force vanifhes, that is, when the motion becomes uniform, the fpace defcribed, $ABSHCA$, and therefore the time, will be infinite.

LECTURE

LECTURE IV.

1. WATER is raiſed in pumps by the preſſure of the air upon the external water.

The common ſucking pump is formed with two ſuckers, each having a valve which opens upwards; the lower ſucker is fixed, the upper moveable; and the water is diſcharged through an orifice above the moveable plug or piſton.

The forcing pump has two ſuckers, the upper of which is moveable, and is a ſolid plug without a valve; the lower is fixed, with a valve opening upwards; and the water is diſcharged through an orifice juſt above the lower ſucker.

2. Water cannot be raiſed by a ſingle ſucking pump to a greater height than thirty-two feet; but if a ciſtern be placed there to receive the water, and another pump work in it, the water may be raiſed to the height of thirty-two feet more, diminiſhed by the height of the column of water which

which balances a column of air of thirty-two feet; and ſo on.

3. The effect of the forcing pump is not limited to the raiſing of water to any particular altitude.

Becauſe the air's condenſation may be increaſed to any degree.

4. If g be the greateſt and l the leaſt altitude of the piſton, of a ſucking pump, above the ſurface of the water in the reſervoir, and h the height of a column of water equivalent to the weight of the atmoſphere; the aſcent of the water by one ſtroke will be $= \frac{g+h}{2} \pm \sqrt{\frac{\overline{g+h}^2}{4h} + l - g}$.

The length of the ſtroke is the difference between the greateſt and leaſt altitude of the piſton.

If y be the aſcent of the water by the firſt ſtroke, ſubſtitute $l-y$ inſtead of l, and you will have the aſcent by the ſecond ſtroke; and ſo on.

5. No pump can raiſe water, unleſs the length of the ſtroke be greater than the ſquare of the greateſt height of the piſton in feet, divided by 128.

Let x be the higheſt aſcent of the water, then $g-x : l-x :: h : h-x$; whence $x^2 - gx = \overline{l-g} \times h$, and $x = \frac{1}{2} g \pm \sqrt{\frac{g^2}{4} + \overline{l-g} \times h}$, which is an impoſſible quantity when $\overline{l-g}$ is greater than $\frac{g^2}{4h}$; that is, x cannot be leſs than l, the leaſt altitude of the piſton.

6. If the length of the ftroke in a uniform pump, which is requifite to render the machine effectual, be greater than can be conveniently made, it may be diminifhed, by contracting the diameter of the fucking pipe in the fubduplicate ratio of the diminution of the length of the ftroke.

7. The velocity of the water flowing from the fucking pipe into the barrel, fhould be equal to the velocity with which the pifton moves.

Otherwife, if it be greater, lefs work will be done than the pump is competent to effect; or if it be lefs, a vacuum will be produced below the pifton, which will therefore be moved upwards with great difficulty.

If V be the velocity of the water in the fucking pipe, d the diameter of the fucking pipe, D the diameter of the barrrel, and v the velocity of the pifton V; then $V \times \frac{d^2}{D^2}$ will be the velocity of the water in the barrel, and $V \times \frac{d^2}{D^2} = v$, when the machine is perfect.

If h be the weight of a column of water whofe weight is equivalent to the preffure of the atmofphere, b the height of the water in the fucking pipe, x any other height to which it afcends in following the pifton, $g = 16\frac{1}{12}$ feet; then will the moving force $= h - x$, the quantity of matter moved $= x$, therefore the accelerating force $= \frac{h-x}{x}$; therefore $vv = 2g \times \overline{\frac{hx}{x} - x}$; and v, the velocity of the water,

water, equal to $\sqrt{4g \times b.\ \text{Hyp. Log.}\ \dot{x} - x}$; but when $x = b$, $v = 0$, therefore the fluent corrected will be $=$ $\sqrt{4g \times b.\ \text{Hyp. L.}\ \frac{x}{b} - \overline{x - b}}$. Ex. gr. ſuppoſe $b = 32$, $b = 16$, $x = 18$, or the length of the ſtroke of the piſton equal to two feet; then $v = 10\frac{1}{4}$ feet. Belidor and Deſaguliers calculate the velocity in a very different manner; and their concluſion is as different.

8. If an horizontal tube having two valves opening horizontally outwards in contrary directions, perpendicularly to the axis of the tube, communicate with another tube which is vertical, and has a valve at the bottom, opening upwards; and the whole machine being filled with water and placed in a ciſtern, it be turned ſwiftly round on a pivot at the bottom; the water will continue to be diſcharged in an uninterrupted ſtream.

This is the centrifugal pump. It is evident that the velocity of the ſtream can never exceed the velocity which a heavy body would acquire in falling down the difference between thirty-one feet, and the height of the vertical tube; nor can this pump raiſe water higher than the common ſucking pump.

9. The diſcharge of water in forcing pumps is rendered continual by the elaſtic force of condenſed air.

Since the ſame quantity of water is diſcharged in the

ſame time, whether an air veſſel be uſed or not, and the flux is conſtant in the former caſe, and interrupted half the time in the latter, the velocity of the ſtream will be but half that in the former caſe; and therefore the machine will ſuffer a leſs ſtrain; from the continuance of the ſtream alſo the water can be directed with greater certainty to any particular place.

LECTURE

LECTURE V.

1. IF there be a ſucking pipe with a valve opening upwards at the top, communicating with a cloſe veſſel of water, not more than thirty two feet above the level of the reſervoir; and the ſteam of boiling water be thrown on the ſurface of the water in the veſſel, it will force it to a height as much greater than thirty two feet, as the elaſtic force of ſteam is greater than that of air; and if the ſteam be condenſed by the injection of cold water, and a vacuum thus formed, the veſſel will be filled from the reſervoir by the preſſure of the atmoſphere; and the ſteam being admitted as before, this water will be alſo forced up; and ſo on ſucceſſively.

This was the principle of the firſt ſteam engine invented by the Marquiſs of Worceſter, and afterwards carried into effect by Mr. Savery.

2. If

2. If the ſteam be admitted into the bottom of a hollow cylinder, to which a ſolid piſton is adapted, the piſton will be forced upwards by the difference between the elaſtic forces of ſteam and common air; and the ſteam being then condenſed, the piſton will deſcend by the preſſure of the atmoſphere, and ſo on ſucceſſively.

This is the principle of the ſteam engine firſt contrived by Meſſeurs Newcomen and Cowley of Dartmouth. This engine is commonly a forcing pump, having its rod fixed to one end of a lever, which is worked by the weight of the atmoſphere upon a piſton at the other end, a temporary vacuum being made below it by ſuddenly condenſing the ſteam, that had been admitted into the cylinder, in which this piſton works, by a jet of cold water thrown into it. A partial vacuum being thus made, the weight of the atmoſphere preſſes down the piſton, and raiſes the other end of the ſtraight lever, together with the water from the well. Then immediately a hole is uncovered in the bottom of the cylinder, by which a freſh quantity of hot ſteam ruſhes in from a boiler of water below it, which proving a counterbalance for the atmoſphere above the piſton, the weight of the pump rods, at the other end of the lever, carries that end down, and raiſes the piſton of the ſteam cylinder. The ſteam hole is then immediately ſhut, and a cock opened for injecting the cold water into the cylinder of ſteam, which condenſes it to water again, and thus making a vacuum below the piſton, the atmoſphere again preſſes it down,

down, and raifes the pump rods as before; and fo on continually.

3. If c be the diameter of the cylinder of a fteam engine, in inches, then will $6c^2$ be the meafure of the power of the machine, in pounds,

The pifton does not defcend with a force exceeding eight or nine pounds upon every fquare inch; but upon account of friction and alterations in the denfity of the air, Doctor Hutton thinks it fafeft, in calculating the power of the cylinder, to allow only 7lb. 10 oz. for every fquare inch, or 6lb. for every circular inch.

4. If p be the diameter of the pump, and f the depth of the pit in fathoms, then will $3c^2 = p^2f$.

For the fection of the pump in circular inches $= p^2 \times$,7854, and $p^2 \times .7854 \times 72 =$ the number of cubic inches in one fathom or fix feet of the pump; but there are 282 cubic inches in a gallon, therefore $\frac{p^2 \times .7854 \times 72}{282}$ $= \frac{1}{5}p^2$ is the number of gallons in one fathom of the pump; now a gallon weighs $10\frac{1}{5}$ lb, therefore $\frac{1}{5}p^2 \times 10\frac{1}{5}$ lb. $\times f =$ the weight of the water in the pump, computed in pounds, which muft be equal to $6c^2$, the power of the engine; that is, $\frac{1}{5}p^2 \times 10\frac{1}{5} \times f = 6c^2$, or $2p^2f = 6c^2$, nearly. See Hutton's Math. Dict.

5. If c be the diameter of the cylinder, and l the length of the ftroke, b the diameter of the boiler ought to be $= \sqrt[3]{30\,lc^2}$.

For the quantity of fteam confumed at each ftroke $=$ lc^2,

k^2; and ſince the top of the boiler is hemiſpherical, its contents will be $= \frac{b^3}{3}$; but if the top contain about ten times the quantity of ſteam uſed each ſtroke, it has been found, that it will require no more fire to preſerve its elaſticity than is ſufficient to keep the water in a proper ſtate of boiling, therefore $10\, k^2 = \frac{b^3}{3}$, and $b = \sqrt[3]{30\, k^2}$.

6. If the cylinder and piſton be ſurrounded at a ſmall diſtance with another cylinder, furniſhed with a bottom and a lid with a hole for the piſton rod to ſlide in; and the interſtice between the cylinders communicate with the ſteam veſſel; and two valves be fixed in the bottom of the inner cylinder, one of which admits the ſteam to paſs from the interſtice into the inner cylinder below the piſton, or ſhuts it out at pleaſure, and the other opens or ſhuts a pipe which communicates with a veſſel called the condenſer, which is alternately a vacuum and filled with ſteam; in ſuch a machine the piſton will be forced downwards by the action of the ſteam above it, when a vacuum is made below it, and upwards by the weight of the pump rods and buckets, when the ſteam being admitted below the piſton, counteracts the preſſure of the ſteam above it.

This is the principle of Mr. Watt's ſteam engine. The advantages of this conſtruction are 1 ſt, that the cylinder being

being ſurrounded by the ſteam from the boiler, is kept always uniformly as hot as the ſteam itſelf; and therefore no ſteam is loſt, as in the other engines, by its contact with a cold piſton. 2. The condenſer being kept always as cold as water, the ſteam is perfectly condenſed. 3. The pump rods and water are raiſed by the preſſure of ſteam, which is greater than that of the atmoſphere.

PNEUMATICS.

LECTURE I.

1. WIND is a current of air.

That part of Natural Philofophy which treats of wind is properly called Pneumatics.

2. The caufes of wind are various, as exhalations, the melting of fnow and ice, electricity, cold, and particularly heat.

3. If the fpring of the air be weakened in any place more than in the adjoining places, a wind will blow through the place where the diminution is.

4. Winds blow into rarer air, out of a place filled with denfer.

5. If the air be fuddenly condenfed in any place, a wind will blow through it.

Becaufe its fpring will be fuddenly diminifhed: this happens

pens when the air in any place has been very much rarified, and then is ſuddenly cooled.

6. If air be ſuddenly rarified, its ſpring will be ſuddenly increaſed; it will therefore flow through the air not acted on by the rarifying force.

7. The trade winds are cauſed by the diurnal revolution of the earth, and the great heat of the torrid zone.

The trade winds are thoſe North Eaſt and South Eaſt winds, which prevail in the northern and ſouthern hemiſpheres throughout the year.

8. The general trade wind does not invariably take place beyond the 28th. or 30th. degree of latitude.

9. The ſea and land breezes of the torrid zone are cauſed by the greater heat, which the land acquires, during the day, than the ſea; and the contrary during the night.

Theſe breezes are gentle periodical winds, regularly ſhifting twice every day, and blowing from the ſea towards the land during the day; and from the land towards the ſea, in the night.

10. The monſoons are cauſed by the compoſition of the general trade wind with thoſe winds, which would be produced by the alternate heating and cooling of the neighbouring ſeas and continents.

The monſoons are thoſe periodical winds, which for ſix months together blow in one direction, and for the

 other

other fix months of the year, in the contrary direction. They prevail in in the northern and part of the fouthern region of the Indian Ocean.

11. The variations of the wind do not arife from the influence of the moon.

12. The velocity of wind in a ftorm is about fixty miles in an hour; twenty miles in a very brifk gale; ten in a pleafant brifk gale; and five in a gentle pleafant wind.

13. The force of the wind upon a fquare foot is about eighteen pounds avoirdupois in a ftorm; two pounds in a very brifk gale; half a pound in a pleafant brifk gale; and two ounces in a gentle pleafant wind.

14. The force of the wind on the fail of a windmill is in a ratio compounded of the duplicate ratios of the fine of incidence and of the velocity of the wind, and the fimple ratio of the area of the fail.

15. The fails of a windmill fhould form an angle of 54°. 44′ with their common axis.

This angle however is only that, which gives the wind the greateft force to fet the fail at firft in motion: but if the machine be already in motion, the angle of incidence muft be varied according to the degree of that motion; for when the fail has a certain motion it yields to the wind; and then that angle muft be increafed to give the wind its full effect. Maclaurin in his Fluxions, p. 734, and in his View of Newton, B. 2. c. 3. has fhewn how to determine this angle

angle. See alſo Saunderſon's Flux. p. 15. And ſince the velocity of the ſails at the very axis is nothing, and increaſes from thence to their extremity, the angle at the very axis ſhould be 54° 44′, and thence continually increaſe to the extremity, giving the vane a twiſt, and cauſing all its ribs to lie in different planes. This is exemplified in the wings of birds.

16. The width of a rectangular ſail ſhould be double its length.

This is demonſtrated by M. Parent; whence he ſhews, that the uſual form, in which the length is generally five times the width, is extremely diſadvantageous.

17. The beſt form of the ſail is an elliptical ſector, whoſe curvilineal ſides converge to the axletree of the mill.

LECTURE

ACOUSTICS.

LECTURE I.

1. AIR is the principal medium of ſound.

This is univerſally admitted: but ſome hold that it is not the only medium, for that water is a medium alſo; in confirmation of which it has been obſerved, that fiſhes appear to be furniſhed with an organ of hearing.

2. Sound is cauſed by the vibration of elaſtic bodies.

Philoſophers are agreed in this, becauſe ſounding bodies communicate tremors to diſtant bodies.

3. If

3. If a tended elastic fibre be drawn from its rectilineal position by any inflecting force perpendicular to the axis, the accelerating force of any particle of the string will be directly as its distance from the axis of rest.

Fig. 1. Let *AB* be an elastic string fixed at the point *A*, and tended by the weight *P* passing over a pulley at *B*; and let it be drawn into the position *AIB* by an inflecting force acting at *I*, in the direction *IR* bisecting the angle *AIB*; take *IF*, *IE* severally equal to half the length of the string *AB*, which is not supposed to be increased by the inflection, and complete the parallelogram *EIFR*; the constant and equal tending forces of the string in the directions *IE*, *IF*, being represented by these sides *IE*, *IF*, they composed a force = *IR* = the inflecting force in the direction *IR*; through *I* draw *IH* parallel to the axis *AB*, and from *R* let fall the perpendicular *RG* on *IH*; the inflecting force in the direction *IR* is to the inflecting force in the direction *RG*, perpendicular to the axis, as *RI* to *RG*; therefore the inflecting force perpendicular to the axis, is to the tending force in the direction *IB*, as *RG* to *IF*, that is, very nearly as 2 *GK* to *IF*, because the angle *IAD*, which is indefinitely little by the hypothesis, is greater than *IEF*; and *IEF* or *IFE* is greater than *FDB* or *ADE*; therefore *IAD* is greater than *ADE*; therefore *DE* very nearly coincides with *DA*; and *GK* : *GR* : : *IC* : *IR* : : 1 : 2; therefore the inflecting force, in a direction perpendicular to the axis, is very nearly directly as *GK* or *IV*, the perpendicular distance of the point *I* from the

the axis of reſt; but the accelerating force at I is equal to the inflecting force.. This is alſo confirmed by experiment.

4. All the parts of a vibrating ſtring perform their vibrations in the ſame time, according to the law of a cycloidal pendulum.

5. All the vibrations of an elaſtic fibre are performed in equal times.

6. The accelerating force of any particle of a muſical ſtring is = the product of the tending force × length, divided by the weight of the ſtring × the radius of the ſtring's curvature in the particle.

Fig. 2. In a muſical ſtring AIB, let IK, LK repreſent two elements of the curve; and let IM, LM be erected perpendicular to theſe elements; they will meet in M the centre of curvature; complete the parallelogram $IKLN$, the tending force of the weight P : motive force of the point K : : IK : KN; but the fibre is every where equally ſtretched by the weight P, therefore the tenſions IK, KL are equal, and the triangle KIN is iſoſceles; then, becauſe of the right angles KIM and KLM, $LKI + M =$ two right angles $= LKI + KIN$, $\because KIN = M$, and the triangles LMI, KIN are ſimilar; and $IK : KN :: IM : IL ::$ tending force P : motive force of the element $IL = \frac{P \times IL}{IM}$; let this force $= A$; alſo let the weight of the ſtring $= W$, its length $= L$; the weight of the element $IL = \frac{IL \times W}{L}$,

let

let this weight $= B$, then the accelerating force of the element $IL = \frac{A}{B} = \frac{P \times L}{W \times IM}$.

7. In a mufical ftring the ordinates are inverfely as the radii of curvature at the extremities of the ordinates of the curve, which the ftring affumes in vibrating.

For fince P, L, and W are given, the accelerating force is inverfely as IM the radius of curvature; but the accelerating force is alfo directly as the ordinate IV, or the diftance of the point from the axis of reft.

8. If there be defcribed two concentrical circles, the radius of the interior being indefinitely lefs than that of the exterior; and, the femidiameter of the interior being confidered as an abfciffa, if an ordinate move from its extremity to the centre fo, that this ordinate fhall be always equal to the arch of the exterior circle, fimilar to that of the interior which the ordinate meets, the extremity of the ordinate will defcribe the harmonic curve.

Fig. 3. For let C be the common centre of the two circles, and the ordinate HI always equal to the arch EG, fimilar to the arch DF; draw WN indefinitely near to KI; from I, F, and O let fall perpendiculars on the axis AB of the curve; draw the radii CFG, COZ; join EL, Ey; and from I and N erect the perpendiculars IM, NM, which meet in M the centre of curvature of the element IN; and with EL as a radius defcribe the arch Lr. Then

$CL : CF :: FP : FO$ (from sim. triangles) and $CF : CE :: FO : GZ$; therefore $CL : CE :: FP$ or $IT : GZ$ or TN, and therefore the triangles CEL, INT, are similar. Now $FL : CL :: PO : PF$, and $CL : EL :: IT$ or $FP : IN$, therefore $FL : EL :: PO : IN$; but $PO = Ly = Lr$, therefore $FL : EL :: Lr : IN :: EL : IM$; whence FL or $IV = \frac{EL^2}{IM} = \frac{EC^2}{IM}$; that is, because EC is constant, the ordinate IV is inversely as IM the radius of curvature; and therefore the curve is that which a musical string assumes in vibrating.

9. The rectangle under the radius of curvature and the ordinate in any point is to the square of the length of the string, in a duplicate ratio of the diameter of a circle to its periphery.

For $IM \times IV = CE^2$; but $CE : 2CB$ or L the length of the string :: radius : semi-circumference :: diameter : p the periphery; and therefore $CE^2 = L^2 \times \frac{\text{diam.}^2}{p^2}$.

10. If W be the weight of a musical string, L its length, P the tending force, and g the space described by a falling body in 1″, then will T the time of vibration of the string $= \sqrt{\frac{LW}{2gP}}$.

Let p be the periphery of a circle whose diameter $= 1$, $F = \frac{P \times L}{W \times IM}$ = the accelerating force of the particle I of the string, at the distance IV from the axis of rest, then

T,

T, the time of the whole vibration, $= \sqrt{\frac{p^2.IV.IM.W}{2g.L.P.}}$

(See Art. 7. Lect. 22. Mech.); but $IV \times IM = \frac{L^2}{p^2}$; therefore, by substitution, $T = \sqrt{\frac{W.L}{2g.P}}$.

11. In homogeneal strings, since their weights are as the squares of their diameters $\times$ the lengths, or as $D^2 \times L$, the times of the single vibrations of the strings will be as $\frac{D \times L}{\sqrt{P}}$; that is, in a ratio compounded of the direct simple ratios of their diameters and lengths, and the inverse subduplicate ratio of their tensions.

12. The tone of a sounding body depends on the number of vibrations which it performs in a given time.

For whenever the time of vibration of two strings is the same, the tone is likewise the same; and whenever the time is different, the tone is so likewise, and that in a certain relation to the number of vibrations.

13. A sound is acuter, sharper, higher than another, if the sounding body perform a greater number of vibrations in a given time.

14. The time of vibration of any sounding body is equal to the time of vibration of an unisonal string.

 15. A

15. A body which gives the graveſt harmonic ſound, vibrates twelve times and an half in 1″; and the ſhrilleſt ſounding body vibrates 51100 times in a ſecond.

16. A ſound is muſical, if the vibrations of the ſounding bodies be iſochronous.

The difference between a muſical ſound and a noiſe is, that a noiſe is a compound of many different and diſcordant tones confuſed together; whereas a muſical ſound is the effect of iſochronal vibrations.

17. The parts of muſical bodies vibrate according to the law of a cycloidal pendulum.

Becauſe they may be conſidered as compoſed of elaſtic fibres.

18. Sounding bodies propagate their motions on all ſides, in directum, by ſucceſſive condenſations and rarefactions, and ſucceſſive goings forward and returnings backward of the particles of air.

Thoſe parts of the air which vibrate backwards and forwards, and which by going forwards ſtrike againſt obſtacles, are called Pulſes.

19. The velocity of ſound is 1142 feet in a ſecond.

This is determined by experiment.

20. All pulſes move equally faſt.

This alſo appears from experiment.

21. The pulſes of air are propagated from ſounding

ing bodies according to the law of a cycloidal pendulum.

For the pitch of a founding body does not alter while the hearer varies his diftance from it; therefore the larger and fmaller vibrations of the particles of air, at fmaller and greater diftances from the founding body are ifochronous; and confequently the accelerating forces of the particles are every where proportional to the little fpaces which they defcribe, as in a pendulum.

22. The number of pulfes propagated is the fame with the number of vibrations of the founding body.

23. The latitude of a pulfe is equal to the fpace which the pulfe defcribes in a given time, divided by the number of vibrations performed in the fame time by the founding body.

24. The decay of found according to the diftance, arifes principally from the want of perfect elafticity in the air.

25. The augmentation of found in fpeaking trumpets depends on its reflection from the tremulous fides of the tube.

26. All points of obftacles which produce an echo, muft lie in the furface of an oblong fpheroid, generated by the revolution of an ellipfe, whofe major axis exceeds the interval between the foci, by a greater fpace than 127 feet.

It

It is not absolutely necessary, that the reflecting points should lie accurately in the surface of this spheroid: if the sums of the lines drawn from the hearer and sounding body to the reflecting objects do not differ from each other by more than 127 feet, the pulses propagated by reflection from those obstacles will not be distinguishable.

27. If there be different echoing spheroids, there will be different successive echoes of the same original sound.

LECTURE II.

1. A CONSONANCE is a varied tone, generated by the simultaneous founding of two mufical bodies, accompanied with a pleafant fenfation.

2. The agreeable fenfation of confonances is not the refult folely of the frequent coincidence of the pulfes.

For if a bafe ftring perform four vibrations while another performs feven, their tones will be difcords; and if there be another ftring which performs eight vibrations while the bafe performs five, the bafe and this latter ftring will be concords. Now while the bafe performs twenty vibrations the firft treble will perform thirty-five, and there will be five coincident pulfes; and while the fame bafe performs twenty vibrations, the fecond treble will perform thirty-two vibrations, and there will be but four coincident pulfes. So that

that in the same time there will be fewer coincidences in the vibrations which are concordant, than in those which are dissonant.

3. The agreeable sensation of consonances depends on the simplicity of their cycles of times.

Fig. 4. The cycle of times of two musical strings, is the series of portions of time between the successive pulses of both strings interchangeably succeeding one another, in beating upon the ear, terminated at both ends by coincident pulses: and this cycle will be more simple, the less the sum of the vibrations of the two strings, and the less the equal times between the pulses of the acuter sound are interrupted and subdivided by the pulses of the graver. For example, let *AB* and its parts represent the time in which a string performs two vibrations, and an equal line *DE* and its three equal parts, the time in which another performs three vibrations, and let the pulses be coincident at the moments of time *A*, *D*, and *B*, *E*; between the first pulse and the second, that beat on the ear, there elapses the time *DF*; between the two next impulses there will elapse the time *Fc*, then *cG*, and lastly *GE*; now this series of times, *DF*, *Fc*, *cG*, *GE* is the cycle of times. Suppose again, that *MN* represents the same time in which another string vibrates once, the sum of the vibrations performed in the given time by *MN* and *DE* together will be less than the sum of the vibrations of *AB* and *DE* in the same time; and therefore the cycle will on that account be simpler. Further, let *PQ* and its four equal parts represent the four vibrations of another string performed in the same time; the

the fum of the vibrations of PQ and MN is equal to the fum of the vibrations of AB and DE in the fame time; but the vibrations of PQ are not fubdivided by thofe of MN, whereas the middle vibration of DE is fubdivided by a pulfe of AB; hence the former cycle of times is fimpler than the latter.

4. If the times of vibration of the two mufical bodies be as one to two, they generate a tone which is called an octave.

5. The octave is not fuppofed to alter the nature of any confonance.

Becaufe none of the equal times between the pulfes of the acuter found are interrupted or fubdivided by the pulfes of the graver.

6. The tone of any ftring and of its half may be confidered as the limits of all poffible tones.

Becaufe we may reduce within thefe limits the tones of all ftrings longer than the whole, or fhorter than the half, by taking the double, quadruple, octuple, &c. of thefe ftrings in the latter cafe; and the fubduple, fubquadruple, fuboctuple, &c. in the former.

7. If unity be fuppofed equal to the length of any ftring, all poffible tones may be reprefented by this unit and all the fractions which are comprifed between unity and its half.

It appears therefore that all the tones, which can be concords to that of a given ftring, are produced by ftrings whofe lengths are intermediate between that of the whole

ſtring and its half; and hence the octave is called the Diapaſon.

8. If two chords perform their vibrations in the ſame time, they generate an unvaried tone called an uniſon.

9. If the times of vibration of two chords be as one to three, they generate a conſonance which is called a fifth or diapente.

10. A ſyſtem of ſounds, in which the muſical primes are one and two only, is rejected for its too great ſimplicity.

For the octave is ſcarcely diſtinguiſhable from its fundamental; hence therefore other conſonances muſt be introduced: now the ratio of 1 to 3 is next, in order of ſimplicity, after the ratio of 1 to 2.

11. By admitting the number three amongſt the muſical primes, beſides a fifth a perfect fourth is likewiſe introduced, which is a conſonance generated by the ratio of 3 to 4.

12. If the times of vibration of two chords be as one to five, they generate a perfect conſonance called a ſixth.

13. From the perfect conſonances and their elements ariſe all the concinnous and inconcinnous intervals.

Thus the difference between a perfect third and perfect fourth $= \frac{4}{5} \div \frac{3}{4} = \frac{16}{15}$, which is the ratio of a hemitone major;

jor; the difference between a perfect fourth and fifth $= \frac{3}{4} \div \frac{2}{3} = \frac{9}{8}$, which is the ratio of a major tone; the difference between a perfect fifth and sixth $= \frac{2}{3} \div \frac{3}{5} = \frac{10}{9}$, is the interval of a minor tone. Also, the difference between a major and minor tone $= \frac{9}{8} \div \frac{10}{9} = \frac{81}{80}$, is the interval of a comma, or schism; the difference between a minor tone and a hemitone $= \frac{10}{9} \div \frac{16}{15} = \frac{25}{24}$, is the interval of a diesis or minor hemitone; the difference between a hemitone major and a hemitone minor $= \frac{16}{15} \div \frac{25}{24} = \frac{128}{125}$ is the interval of the enharmonic diesis, or minor diesis, or quarter of a note; and the difference between a hemitone major and a comma $= \frac{16}{15} \div \frac{81}{80} = \frac{256}{243}$ is the interval of a limma. This latter interval was anciently called the Pythagorean Hemitone, because the Pythagoreans did not use the minor tone; and therefore their fourth, being composed of two major tones and a complete hemitone, would have been too great by a comma; they were therefore obliged to diminish the hemitone by this quantity.

14. If no other primes than 1, 2, 3 were admitted into the composition of consonances, a system of sounds thence resulting could have no perfect thirds, nor any perfect consonance whose vibrations are in any ratio having the number five, or any multiple of it, for either of its terms.

15. If in a system of sounds which admits no other primes than 1, 2, 3, we ascend by a perfect

fifth,

fifth, and defcend by a perfect fourth alternately, the octave will be divided into five major tones and two limmas.

Fig. 5. Becaufe the difference between thefe fucceffive fourths and fifths are major tones; and the difference between three major tones and a fifth, and between two major tones and a fourth is a limma.

16. In fuch a fyftem, the confonances that would be perfect, if the number five were admitted amongft the mufical primes, will be produced by powers of eight and nine; the terms therefore of the ratios being fo high, thefe confonances become extremely difagreeable; and confequently this fyftem muft be rejected.

17. If the number five be admitted, which gives the ratio next in order of fimplicity after the ratio of 1 to 3, the octave will be divided into three major tones, two minor tones, and two hemitones.

Fig. 6. Let E be a perfect third major, F a fourth, G a fifth, and A a fixth, to the fundamental C; from E to F is an interval of $\frac{15}{16}$, which is called a hemitone, and denoted by the letter H; from F to G is an interval of $\frac{8}{9}$, which is called a tone major, and denoted by the letter T; from G to A is an interval of $\frac{9}{10}$, which is called a tone minor, and is denoted by the letter t. Now the lengths of the ftrings C, E are to each other in the fame ratio as the lengths of F and A, and therefore they comprife the interval of a tone

tone major and tone minor. Alſo, the ſtrings *A*, *c*, are to each other in the ſame ratio as *E* and *G*, and conſequently compriſe the ſame intervals, to wit, a tone major and an hemitone; and thus the elements of the octave are three major tones, two minor tones, and two hemitones.

18. If we aſcend by the perfect intervals there will ariſe the major mode.

For a perfect third, fourth, fifth and ſixth, being tuned to the fundamental, the interval between the third and fourth is an hemitone; and the interval between the fourth and fifth is a tone major; and the interval between the fifth and ſixth is a tone minor; but the interval between the fundamental and the third is equal to the interval between the fourth and ſixth, that is, equal to a major and a minor tone, and therefore is a major third; from which the mode is denominated.

19. If a muſical ſtring and its parts be in proportion to one another as the numbers 1, $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, $\frac{1}{2}$, their vibrations will exhibit the natural ſeries of ſounds in the octave. Fig. 7.

Fig. 6. For, a perfect third, fourth, fifth and ſixth being tuned to the fundamental, the order of their elements is as natural as that of the concords themſelves; therefore in inſerting a tone between the baſe note and the third, and between the ſixth and the octave, it muſt be done ſo, that the ſame order may be preſerved; that is, the interval between the baſe note and the next muſt be a tone major, and of conſequence, the complement to the third a tone minor; and the interval next the octave muſt be that of an hemitone,

hemitone, and of courſe the interval below it a tone major; for by this arrangement, the hemitone of the preceding octave will conſtitute the natural order of the elements with the next major and minor tones of the ſubſequent octave; and any other mode of inſertion will diſturb this natural order. Thus we ſee that the arrangement of the elements of the octave is the natural and neceſſary conſequence of the order of the perfect conſonances.

20. If we deſcend by the perfect intervals, there will ariſe the minor mode.

Fig. 8. For in this caſe $c : A :: E : C :: \frac{4}{5} : 1 :: \frac{1}{2} : \frac{5}{8}$, which is therefore the length of A; in like manner $c : G :: F : C :: \frac{3}{4} : 1 :: \frac{1}{2} : \frac{4}{6} = \frac{2}{3}$, which is the length of G; alſo, $c : F :: G : C :: \frac{2}{3} : 1 :: \frac{1}{2} : \frac{3}{4}$, which is the length of F; and $c : E :: A : C :: \frac{3}{5} : 1 :: \frac{1}{2} : \frac{5}{6}$ which therefore is the length of E; hence the fundamental C is to the third $:: 1 : \frac{5}{6}$, which therefore is a minor third; from which the key is denominated.

21. The diatonic ſyſtem is that in which the melody proceeds by tones major and minor, and hemitones.

In this ſyſtem muſicians denote the eight ſounds of the octave, in the natural order, by the letters *C*, *D*, *E*, *F*, *G*, *A*, *B*, *c*; and ſometimes by the ſyllables, *Ut*, *Re*, *Mi*, *Fa*, *Sol*, *La*, *Si*, *ut*.

22. If we aſcend by the ſame perfect intervals by which we deſcend in the major mode, the arrangement of the elements will be equally natural.

Fig.

Fig. 8. The order of the perfect intervals, by taking their reciprocals, will now be $\frac{2}{1}$, $\frac{5}{3}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, 1; or their equivalents 1, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{1}{2}$; but the first interval, from 1 to $\frac{5}{6}$ is a minor third, and is equal to that between $\frac{3}{4}$ and $\frac{5}{8}$, that is, it contains a tone major and an hemitone; and the interval between $\frac{5}{8}$ and $\frac{1}{2}$ is equal to the interval between $\frac{5}{6}$ and $\frac{2}{3}$, and therefore contains a major tone and a minor tone. But in inserting these tones we naturally proceed in such a manner, as that the same elements shall follow each other in the same order, in different parts of the ascent; therefore the major and minor tones will be outermost, so that the tone minor of the preceding octave will compose the natural order with the tone major and hemitone of the subsequent octave; this order being already ascertained thus, *t*, T, H, by the perfect consonances.

23. These two modes are called the major and minor mode.

Because the third above the fundamental in the former is a major third, and in the latter a minor third.

24. There can be no more than two modes.

Because the perfect concords being given, and consequently their order, there can be no other variety than either ascending or descending by them; so that if in ascending in the major mode the complement of the perfect sixth to an octave had been a major third, there would have been no such thing as a minor mode.

25. A system of sounds whose elements are tones

major,

major, and minor, and hemitones, will neceſſarily contain ſome imperfect concords.

Fig. 9. For in both modes there is an imperfect minor third, which is deficient by a comma, or the difference between a tone major and tone minor. Now if you tune upwards from *C* the two perfect fifths *CG*, *Gd*, and the perfect ſeventeenth, or a double octave and third *Cè*; then downwards the fifth *èa*; the intermediate fifth *ad* will be too little by a comma, as including the imperfect minor third *df*. And by tuning an octave below *a*, we ſhall have the imperfect fourth *Ad*, too large by a comma, as conſiſting of two major tones and an hemitone.

26. No voice or perfect inſtrument can always proceed by perfect intervals without erring from the pitch.

This immediately follows from the laſt article. Thus, if a perſon ſing the notes *C*, *F*, *D*, *G*, *C*, alternately aſcending and deſcending by perfect intervals, the latter *C* will be lower than the former by an entire comma; for the ratios of theſe intervals are 4 : 3, 5 : 6, 4 : 3, and 2 : 3; which compoſe the ratio of 160 : 162, or 80 : 81, which is the ratio of a comma.

In like manner on the violin, if the four ſtrings be tuned perfect fifths, the firſt ſtring or treble will ſound the octave to the perfect ſixth above the baſe, but it will be too acute by a comma; for ſuppoſe the number of vibrations of the baſe ſtring, *ſol*, in a given time to be twelve, the next *re* will perform eighteen in the ſame time, the ſecond ſtring *la* 27, and the treble *mi*, 40½, becauſe 2 : 3 is the ratio

ratio of a perfect fifth; but the perfect sixth above *sol* 12, is 20, because the ratio of a sixth is 3 : 5, and 3 : 5 :: 12 : 20, whose octave is 40; whereas the treble is $40\frac{1}{2}$, that is, a comma higher than the perfect octave of *mi*, for 40 : $40\frac{1}{2}$ :: 80 : 81.

27. No other primes than 1, 2, 3, 5, are to be admitted into musical ratios.

First, because the consonances whose vibrations are expressed by terms involving other primes, as 7, 11, 13, &c. would, cæteris paribus, be less simple and harmonious. Secondly, as perfect fifths and other intervals resulting from the number 3 make the schism of a comma with the perfect thirds and other intervals resulting from the number 5, so such intervals as result from the numbers 7, 11, 13, &c. would make other schisms with both those kinds of intervals. See Smith's Harmonics, p. 33.

28. The disagreeable effect of the imperfect concords in the diatonic system has obliged musicians to have recourse to tempered systems.

Fig. 9. The disagreeable effect in every octave of the 5th. *da*, which is deficient by a comma, and of the 4th. *dA*, which is excessive by a comma, has obliged musicians to distribute this comma equally amongst the four 5ths. contained in the double 8ve. and third; or within the compass of 17 notes from the base note: this distribution is called the Participation, or Temperament of the system. See Smith's Harmonics.

29. If the periphery of a circle be divided so as to represent the order and proportion of the ele-

ments of the octave in the diatonic syſtem, whether in the major or minor mode, and the major third which lies between the two hemitones be biſected; and the other two major tones be diminiſhed at both ends by a quarter of the difference between a major and minor tone, the octave will be divided into five mean tones, and two limmas, each limma exceeding an hemitone by the quarter of a comma.

Fig. 10. For the major tone *CD* being diminiſhed by half the difference between it and a minor tone, and the minor tone *DE* as much increaſed, they are rendered equal. Alſo the minor tone *GA*, which lies between the two major tones, is increaſed at each ſide by a quarter of a comma, that is, in all by half a comma; and the major tones, which lie at each ſide of it, are diminiſhed at both ſides by a quarter of a comma, that is, in all by half a comma; and thus all the tones are reduced to an equality. Hence this is called the ſyſtem of mean tones. The limmas alſo are equal by conſtruction.

30. In the ſyſtem of mean tones, every perfect 5th. is diminiſhed by a quarter of a comma.

For every tempered 5th. = 3 mean tones + 1 limma, = 3 minor tones + 1 hemitone + 1¼ comma; but a diatonic or untempered 5th. $= 2\,T + t + H = 3\,t + H + 2$ commas.

This is uſually called the Vulgar Temperament; and Mr. Huygens thought it the beſt.

31. In a ſeries of fixed notes, eight tones in the octave

octave are insufficient, either in tempered or untempered melody.

Fig. 11. In a series of fixed notes, any one of them may be indifferently made the principal, and all the others referred to it; and thus a scale formed similar to the original Diatonic, or Tempered scale. Now as the order of the intervals is different according to the note from which we begin the order, new notes must be inserted, that the due intervals may be preserved: for example, suppose *G* is made the base note, then the series of notes from *G* must be similar to those from *C*; in this case, *E* in the key of *G* will be correspondent to *A* in the key of *C*; now from *A* to *B* is an entire tone; but from *E* to *F* is an hemitone; hence therefore between *F* and *G* there must be inserted a note, expressed by the mark *F*✱, whose distance from *E* may be an entire tone; thus will the interval between *F* and *G* be divided into two, the interval between *F* and *F*✱ being the difference between a tone and a hemitone; and of course, the interval between *F* and *G* equal to a hemitone. The first of these two intervals is called a minor hemitone: it is therefore the interval between two notes of the same name. In the tempered system, this interval is equal to the difference between a mean tone and a limma, and is then called a Minor Limma; in the untempered system it is equal to the difference between a tone minor and a hemitone, its ratio therefore is 24 : 25; for 9 : 10 :: 24 : $26\frac{2}{3}$; but this latter interval is composed of 24 : 25, a minor hemitone; and 25 : $26\frac{2}{3}$ = 15 : 16, a major hemitone.

 32. If

32. If the natural ſcale be ſucceſſively raiſed a $5^{th.}$, there muſt be a ſharp or dieſis added for every ſuch elevation.

Becauſe if we make *Sol*, which is the $5^{th.}$ above *Ut* in the natural ſcale, the key note, the $4^{th.}$ to *Ut*, which is *Fa*, muſt be ſharpened, as was ſhewn in the laſt article. For the ſame reaſon, if we raiſe the ſcale to a $5^{th.}$ above *ſol*, the $4^{th.}$ above *ſol* muſt be ſharpened; and ſo on, until we return to *C ſol, ut*, to which eight ſharps would, by this rule, be added; but to ſharpen every note in the octave makes no alteration in their diatonic relation; and therefore ſuch an addition of ſharps is nugatory, that is, the ſcale remains in its primitive diatonic ſtate.

It is farther evident, that if we would lower the ſcale by one, two, or three fifths, &c. we muſt take away one, two, or three ſharps, &c. but to take away a ſharp is equivalent to adding a flat; therefore if the key note be a fifth below *C ſol, ut* major, one flat or bèmol muſt be added, and ſo on; whence the annexed table is conſtructed, exhibiting at one view all the ſharps and flats of every key, major and minor.

33. The chromatic ſyſtem is that in which the melody proceeds ſucceſſively by minor and major hemitones.

As major hemitones only are found in the diatonic ſyſtem, ſo the minor hemitones characteriſe the chromatic.

The major hemitone ariſes naturally in the diatonic ſyſtem; the minor hemitone is the introduction of art, not of

of nature; for the voice does not naturally form two notes in succession between which this interval subsists.

34. A just hemitone is impracticable in music.

For suppose the ratio of a tone to be 9 : 8, that of the half note must be $\sqrt{9} : \sqrt{8}$, or $3 : 2\sqrt{2}$, which are incommensurable quantities. As to the hemitone major, it exceeds half a tone; and a hemitone minor falls short of it. In like manner a just mean tone is impracticable, for it is half a major third; and therefore its ratio is $\sqrt{5}$ to $\sqrt{4}$, or $\sqrt{5}$ to 2.

35 In an untempered chromatic system, if the voice always proceeds by major and minor hemitones, making all the intervals accurate, the octave will be depressed by three commas.

For we thus substitute a major and minor hemitone, that is, a minor tone, instead of each major tone; but there are three major tones in the octave; and the difference between a major and minor tone is a comma.

36. Any instrument which has but twelve notes in the octave is imperfect.

Fig. 12. For if the diatonic scale be raised a fifth, the fourth above the preceding key note must be sharpened; and if it be depressed a fifth, the fourth above the new key note must be flattened; for example, let *F*, which is the fifth below *C*, be made the key note, then will *A* in the new key be correspondent to *E* in the original key; but from *E* to *F* in the primitive key is but an hemitone, whereas from *A* to *B* is an entire tone; therefore between *A* and *B* there must be inserted a tone denoted by B^b whose

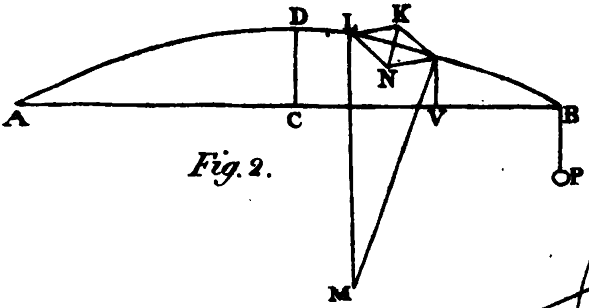
whose distance from A is an hemitone; and consequently its distance from B is the difference between an entire tone and an hemitone, that is, an interval equal to a diesis or sharp; but a diesis, or bemol which is equal to it, is less than half a tone; therefore the same note is not the sharp of the preceding note, and the flat of the subsequent one; and both are necessary.

37. In every instrument with fixed sounds, every equal tone should be divided into two minor limmas with a diesis between them; and each primary limma into two dieses, with an interval between them.

See Smith's Harmonics, Sect. 8. Art. 1.

OPTICS.

Fig. 2.



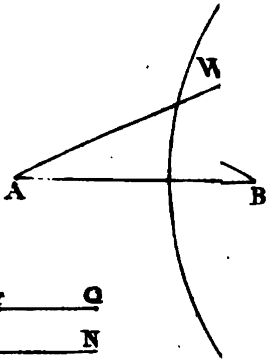
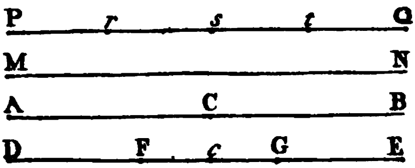


Fig. 4.



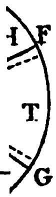


Fig. 7.

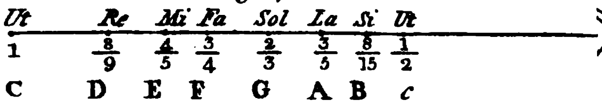
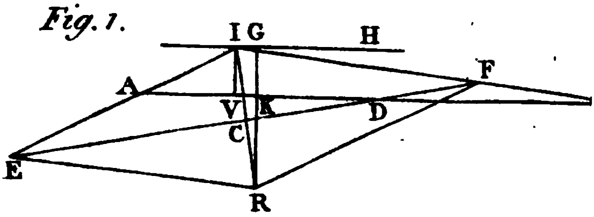


Fig. 1.



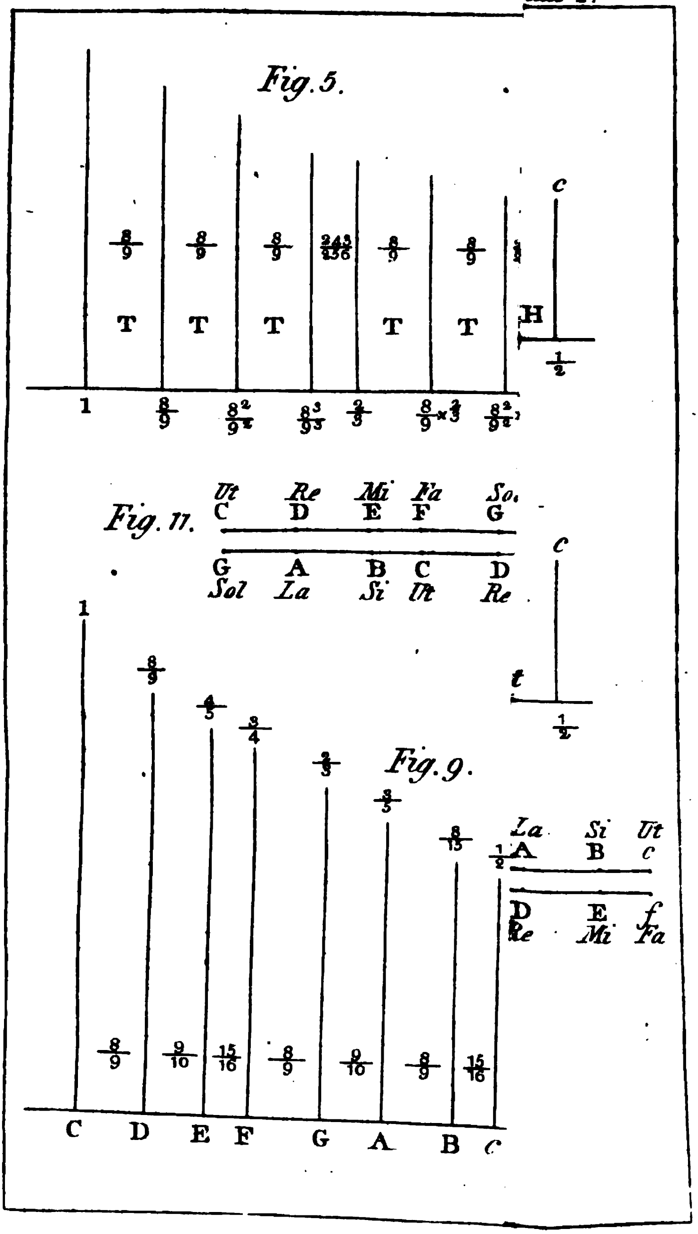


Fig. 5.
T
T
T
T
T
H
c
Fig. 11.
Ut Re Mi Fa Sol
C D E F G
G A B C D
Sol La Si Ut Re
c
t
Fig. 9.
La Si Ut
A B c
D E f
Re Mi Fa
C D E F G A B C

OPTICS.

LECTURE I.

1. OPTICS is that part of Natural Philoſophy which treats of the element of light, and the various phænomena of viſion.

2. It is generally divided into three parts, viz. dioptrics, which treats of refraction; catoptrics, which treats of reflection; and chromatics, which treats of the various phænomena of colour.

3. Many of the ancient philoſophers, of whom Pythagoras ſeems to have been the firſt, rightly conſidered

confidered light as a real emanation of the proper fubftance of the luminous body. Others, on the contrary, afcribed vifion to the iffuing of the luminous particles from the eye to the object.

4. Amongft the moderns there have been two leading opinions concerning the nature of light, the Cartefian and the Newtonian. The Cartefians are of opinion that light confifts in the preffure of a fluid, prefent at all times and in all places, but which requires to be fet in motion by another body properly qualified for that purpofe, which is called luminous. Newton maintains, that light confifts of a vaft number of exceedingly fmall particles, thrown off in all directions from the luminous body.

5. Thefe particles are emitted in right lines by the body from which they proceed.

6. Single particles of light, fucceeding each other in a right line, conftitute a ray of light, confidered in a mathematical fenfe; but, phyfically fpeaking, a ray is the leaft part of light on which alone experiments can be made.

A flender portion of rays feparated from the reft, is called a Pencil of rays.

Pencils of rays are either conical or cylindrical; the axis of the pencil is the fame with the axis of the cone or cylinder.

A Radiant point is the vertex of the cone; and is fo called in

in a general ſenſe, whether the rays diverge from, or converge to it.

7. The denſity of light, abſtracting from any obſtruction it may meet with in its progreſs, decreaſes in the inverſe duplicate ratio of the diſtance from the luminous body.

8. Bodies that ſhine with a native light, are brighter than opaque bodies illuminated by them.

If a luminous body be equally diſtant from an opaque body and from the eye, the light received by the pupil directly from the lucid body is to that which it receives from the opaque body, on ſuppoſition that it reflects all the light incident upon it, as the hemiſphere whoſe radius is the diſtance of the eye from the opaque, to the real diſc or great circle of the opaque body. Hence, on the ſame hypotheſis, day-light is to moon-light nearly as 96000 to 1.

9. The apparent diameter of a body is inverſely as its diſtance from the ſpectator's eye.

Hence the ſuperficial apparent magnitude of an object is inverſely as the ſquare of the diſtance.

10. Whatever grants a paſſage to light, as any tranſparent body, alſo empty ſpace, is called a medium.

11. The moſt tranſparent known medium, even air, obſtructs the paſſage of light; and this is the cauſe why objects appear leſs bright, the farther they are from the ſpectator; for if none of the rays were ſtopped in their paſſage, the magnitude

 of

of the picture on the retina, and the denſity of light would vary in the ſame ratio; and therefore the denſity of light on the retina, and conſequently the luminouſneſs of the object, would be invariable at all diſtances.

12. Diſtance is chiefly eſtimated by this gradual diminution of light, and by the angle which a body ſubtends at the eye.

13. If the rays of the ſun be tranſmitted through a very ſmall circular aperture into a dark chamber, and received perpendicularly on a ſcreen, they will paint a circular image of the ſun, which image will increaſe according to the diſtance of the ſcreen from the aperture.

14. The diameter of this ſolar image is to its diſtance from the aperture, as twice the tangent of the ſun's apparent ſemi-diameter to radius.

If the diſtance of the image from the aperture be eighteen feet, the diameter of the ſolar image will be about two inches.

The image formed by the ſolar rays which flow through a very ſmall aperture, is termed, for the ſake of diſtinction, the correct image, being of a given magnitude when the diſtance from the aperture is given, and alſo better defined than when the aperture is larger.

15. If the aperture be of a ſenſible magnitude, and of any particular figure, the image will be thus determined; on the ſcreen which receives the ſolar

ſolar rays perpendicularly, deſcribe a plane figure ſimilar and equal to the aperture: let circles equal to the correct image of the ſun at the given diſtance of the ſcreen from the image, be deſcribed from centres coincident with the perimeter of this figure; theſe circles will determine the figure of the image.

Hence therefore the entire figure will not be ſimilar to the aperture, in a geometrical ſenſe, although it will partake of the ſame form, and will approach more nearly to that of the aperture, the nearer the ſcreen is brought to it: as the ſcreen recedes from the aperture, the image approaches to a circular form, with which it ultimately coincides, if the diſtance be increaſed ad infinitum.

16. When the ſolar rays are tranſmitted through a circular aperture of a given magnitude, and are received perpendicularly on a ſcreen, the image will conſiſt of a bright central image, and a penumbral annulus of fainter light ſurrounding it.

Let A be the ſemi-diameter of the aperture, and S the ſemi-diameter of the correct ſolar image correſponding to the diſtance between the aperture and ſcreen; then, A being greater than S, the ſemi-diameter of the central image will be $A—S$; and if the diſtance from the aperture be given, the ſemi-diameter of the bright central image will increaſe ad infinitum, while the aperture is indefinitely increaſed: if the aperture be diminiſhed, the ſemi-diameter of the bright central image will decreaſe till A becomes

equal to *S*, in which case the bright central image vanishes; and if it be still farther diminished ad infinitum, the bright central image will increase, until its semi-diameter becomes ultimately equal to that of the correct image.

17. The hypothesis of the materiality of light, or of its consisting of extremely small particles emitted from luminous bodies, seems to be sufficiently proved by the phænomena of the Bolognian stone; by those experiments which demonstrate, that the colour and inward texture of some bodies are changed in consequence of their exposure to light; and by its finite velocity.

18. The finite velocity of light was first discovered by Roemer, from observations of the eclipses of Jupiter's satellites; and he shewed, that light takes up about eight minutes in traversing the semi-diameter of the earth's orbit.

19. The successive propagation of light is also farther established by the aberration of the fixed stars.

This aberration is proved to arise from the progressive motion of light compounded with the annual motion of the earth in its orbit; from which composition of motion Mr. Bradley demonstrated, that the fixed stars would appear to describe ellipses, of various magnitudes, according to their declination, whose centres would lie in the true places of the stars. And if the velocity of star light be supposed equal

equal to that of direct solar light, the magnitudes of those ellipses will exactly correspond with phænomena.

20. The two principal objections to the materiality of light are, 1. that as rays of light would on this hypothesis, continually pass in every direction from every visible point, they must necessarily interfere with each other in such a manner as to destroy the distinct perception of objects. 2. By the constant emission of luminous particles, the sun and stars must have been greatly diminished since their first creation; and the mass of the earth and planets sensibly encreased.

21. The first objection is solved by this consideration, that the action of light produces on the eye an effect that is not instantaneous, and therefore it may excite a constant sensation, notwithstanding an interval of many miles exists between the immediately consecutive particles; and consequently abundant room may be afforded for other rays to pass undisturbed in all directions: the second objection is solved by the excessive minuteness of the particles of light.

LECTURE

LECTURE II.

1. CATOPTRICS is that part of optics which treats of the reflection of light, and the various phænomena depending on it.

Reflexibility is the diſpoſition of the rays to be turned back into the medium from whence they came; and the change of motion, which the ray undergoes in this caſe, is called Reflection.

The angles which the incident and reflected rays contain with the perpendicular to the reflecting ſurface at the point of incidence are, called the angles of Incidence and Reflection. Any ſmooth ſurface reflecting light is called a ſpeculum, and is either plane, convex, or concave.

A line drawn through the centres of the ſphere and of the leſſer circle which terminates a ſpherical ſpeculum is called the Axis.

2. The

2. The incident and reflected rays are in the same plane; and the angles of incidence and reflection are equal.

3. Any error in the figure or position of a speculum produces a double error in the effect.

4. Parallel rays falling on a reflecting plane are reflected parallel.

Parallel rays are such as proceed equally distant from each other through all their course. Rays proceeding from any point, and as they proceed, receding farther asunder, are called Diverging, and such as tend towards a certain point at which they would at last unite, if not prevented, are called Converging rays.

5. If rays diverging from a radiant point fall on a plane speculum, the focus of the reflected rays will be in the perpendicular let fall from the radiant point on the reflecting surface, and as far behind it as the radiant is before it.

The focus is that point from which rays diverge, or towards which they converge. The point from which rays seem to diverge, when in reality they diverge from another, is called a Virtual focus. The focus before reflection is called the focus of incident rays; and the focus after reflection is called the focus of reflected rays; and both together are called conjugate foci, which of consequence are two such points, that either of them being the radiant point or focus of incident rays, the other will be the focus of reflected rays.

6. If

6. If a radiant object be parallel to a plane speculum, the image formed by reflection will be likewise parallel to it, but transposed; of the same size with the object; and at the same distance behind the speculum, that the object is before it.

7. If a person view himself in a plane speculum placed upright, he will see his image complete in a part of the speculum, whose length and breadth is equal to half the length and breadth of the corresponding parts of his own body.

8. If a radiant object be at right angles with a plane speculum, which is parallel to the horizon, the image will be perpendicular to the horizon, but inverted.

9. If a plane speculum be inclined to the horizon in an angle of 45° with its face downwards, the image of a vertical object will be horizontal; and the image of an horizontal object will be vertical.

10. If two plane speculums be inclined to each other in any given angle which is an aliquot part of 360°, the images of the sector contained by the speculums will be all concentric; and the number of these images will be such, as will exactly complete the circle; and therefore if an object be placed between the speculums, the number of images will be equal to the number of sectors necessary

ceffary to complete the circle; and they will appear to ftand round in the circumference of a circle, whofe centre is the point in which the fpeculums concur.

11. If a pencil of parallel rays be incident on a convex fpherical fpeculum, they will be reflected diverging; and the focus of the reflected rays will bifect the radius which is parallel to the incident rays, and be virtual.

The focus of parallel rays is called the principal focus; and its diftance from the fpeculum is called the focal length of the fpeculum.

12. If the point of incidence of parallel rays be at a fenfible diftance from the vertex of the fpeculum, the reflected ray will interfect the axis in a point between the principal focus and the vertex, or middle point of the fpeculum.

The interval between that point and the principal focus is called the longitudinal aberration; and the diameter of the leaft circle into which all the rays can be collected, is called the lateral aberration.

13. If the convex fpeculum be a paraboloid, the rays will be reflected from the focus without any aberration.

14. When diverging rays are incident on a convex fpeculum, they are reflected from a virtual focus, which is found by dividing the radius of the

ſpeculum into two parts, having the ſame ratio to each other, which the diſtance of the radiant point from the centre has to its diſtance from the ſurface of the ſpeculum.

15. If the breadth of the ſpeculum be conſiderable, the rays diverging from the ſame point will not be reflected from the ſame virtual focus in the axis; thoſe rays whoſe points of incidence are more remote from the vertex of the ſpeculum, meeting the axis in a point which is nearer to the ſpeculum.

16. If the incident rays diverge from one focus of an hyperboloid they will be reflected from the other without any aberration, and the focus will be virtual.

17. If a radiant object be placed before a convex ſpeculum, 1. The image will appear behind the ſpeculum. 2. It will appear erect. 3. It will be leſs than the object. 4. As the object approaches the ſpeculum, the image will likewiſe approach the ſpeculum; and increaſe, till at length, when the object touches the ſpeculum, the object and image will meet, and be equal. 5. The image will be convex towards the object.

LECTURE

LECTURE III.

1. IF a pencil of parallel rays fall on a ſpherical concave ſpeculum, they will be reflected converging; and the focus of the reflected rays will biſect the radius that is parallel to the incident rays, and be real.

2. If the breadth of the ſpeculum be conſiderable, the parallel rays will not be reflected to the ſame mathematical point, but will be diffuſed through a little circle, which is called the circle of aberration; thoſe rays whoſe points of incidence are farther from the vertex, meeting the axis in a point which is nearer to the ſpeculum.

3. If the ſpeculum be a paraboloid, the rays will be reflected into the focus without any aberration

4. If rays diverging from an object beyond the centre, fall on a fpherical concave fpeculum, they will be reflected to a real focus in the axis of the fpeculum, which divides its radius into two parts, having the fame ratio to each other which the diftance of the radiant point from the centre has to its diftance from the vertex of the fpeculum.

5. If the breadth of the fpeculum be confiderable, the rays diverging from the fame point will not be reflected to the fame mathematical focus in the axis, thofe rays whofe points of incidence are more remote from the vertex, meeting the axis in a point which is nearer to the fpeculum.

Hence both in this cafe, and where the incident rays are parallel, all the reflected rays will interfect each other between the points of incidence and the axis; and fince the denfity of the rays will be greater near the interfections of the contiguous pencils, there will be formed a luminous curve by this feries of interfections, which is called a Cauftic. If from the radiant there be drawn two tangents to the fection of the fpeculum made through the axis, thefe rays will not be reflected; therefore the Cauftic will be a curve of contrary flexure, whofe cufpis is the geometrical focus, and the extremities of the branches are the points of contact.

6. If the incident rays diverge from one focus of an ellipfoid, they will be reflected to the other focus without aberration; and the focus will be real.

7. If

7. If rays diverging from a radiant placed in the centre, fall on a concave ſpherical ſpeculum, the focus will coincide with the centre, and the image with the object.

8. If the radiant be placed between the centre and the principal focus, the image will be formed at the other ſide of the centre; and the diſtance of the radiant from the ſpeculum will be to its diſtance from the centre, as the diſtance of the focus from the ſpeculum to its diſtance from the centre.

9. If the radiant be placed in the principal focus, the rays will be reflected parallel.

10. If the radiant be placed between the principal focus and the ſpeculum, the rays will be reflected diverging from an imaginary focus behind the ſpeculum; and the poſition of the focus will be determined in the ſame manner as before.

11. The foci of incident and reflected rays are always on the ſame ſide of the principal focus, but on different ſides of either the centre or ſurface.

If the focus of incident rays be moved along the axis of the ſpeculum, the focus of reflected rays will move in the oppoſite direction; and the foci will meet at the ſurface and centre.

12. If the focus of incident rays be moved, its velocity will be to the velocity of the focus of reflected rays,

rays, as the ſquare of the diſtance between the principal focus and the focus of incidence to the ſquare of half the radius.

13. The image is inverted or erect with reſpect to the object, according as they are at different ſides, or on the ſame ſide of the centre of the ſpeculum.

14. The magnitudes of the image and object are to each other as their diſtances from the centre of the ſpeculum; or as their diſtances from the ſpeculum; or as the principal focal length to the diſtance of the image from the principal focus.

Hence if the object be beyond the centre, it will be larger than the image, becauſe farther from the ſpeculum. As the object approaches the centre, ſo does the image; conſequently they approach to equality, and in the centre they coincide. If the object be between the principal focus and centre, the image will be at the other ſide of the centre, and therefore greater, as being farther from the ſpeculum. If the object be between the principal focus and the ſpeculum, the image will be behind the ſpeculum, and therefore greater than the object, becauſe farther from the centre.

15. A rectilineal object placed upright between a concave ſpeculum and its principal focus, will appear concave.

16. The brightneſs of the image formed by a concave ſpeculum is as the area of the ſpeculum directly,

rectly, and the ſquare of the diſtance of the image from the ſpeculum reciprocally.

17. The degrees of heat generated in the foci of different ſpeculums when expoſed to the ſun's rays, are as their areas directly, and inverſely as the ſquares of their focal lengths.

18. The heat generated in the focus of a ſpeculum is to the ſun's direct heat, as the area of the ſpeculum to the area of the image.

LECTURE IV.

1. DIOPTRICS is that part of optics which treats of the refraction of light, and the various phænomena which refult from it.

2. When light moves through an homogeneous medium, it preferves a continued rectilineal courfe; but in paffing from one medium to another of different denfity, it deviates from its former courfe; this change of direction is called refraction.

3. The lines which a ray defcribes before and after it enters the refracting medium are called the incident and refracted rays; the angle contained between the incident ray and a perpendicular to the furface drawn from the point whereon the ray falls, is called the angle of incidence; the angle contained

contained between the refracted ray, and the perpendicular above mentioned is called the angle of refraction.

The difference of the angles of incidence and refraction is the angle by which the ray deviates from its original direction, and is called the Refracted angle, or the angle of deviation.

4. When a ray falls obliquely on the ſurface of a denſer medium, it is refracted towards the perpendicular; a ray falling obliquely on the ſurface of a rarer medium, and paſſing into it, is refracted from the perpendicular; a ray falling perpendicularly on any refracting ſurface, is not turned out of its courſe, but proceeds in the ſame direction with the incident ray.

5. In all refractions between the ſame given mediums, the ſines of the angles of incidence and refraction are to each other in a given ratio

If a ray of light be refracted out of air into glaſs, the ſine of incidence is to the ſine of refraction as thirty-one to twenty, or as three to two nearly; out of air into water as four to three. Hence the ſine of incidence is to the ſine of refraction, when a ray paſſes out of water into glaſs, as $\frac{3}{4}$ to $\frac{20}{31}$, or as 93 to 80.

6. Parallel rays falling on a refracting plane ſurface are refracted parallel.

7. If parallel rays paſs through a denſer me-

dium terminated by two plane ſurfaces inclined towards each other in a very ſmall angle, and the angle of incidence be alſo very ſmall, the angle of deviation contained by the incident and emergent rays will be to the refracting angle contained by the planes, as the difference between the ſines of incidence and refraction at the firſt ſurface to the ſine of refraction.

Fig. 1. 2. Let QA be the incident ray, CS the emergent ray, and let the perpendicular AB to the firſt ſurface AI croſs CD, the perpendicular to the ſecond, in E; and ſuppoſing the ray AC to go both ways out of the priſm, the angle of incidence ACD : angle of emergence DCT, in the given ratio of incidence to refraction, $: : i : r$; and disjointly, $ACD : ACT : : i : r - i$; and $CAB : CAR$ in the ſame ratio, ſuppoſing the ray to go backward; and conjointly or disjointly, $ACD \pm CAB$ is to $ACT \pm CAR$, that is, BEC or AIC is to RFS, in the ſame given ratio of i to $r - i$.

Hence the deviation is proportional to the refracting angle, and is invariable in all poſitions of the ray; and when the ray within the priſm coincides with a perpendicular to either of the ſurfaces, one of the refractions will vaniſh, and the deviation cauſed by the other ſingle refraction will continue the ſame in quantity as before, when it was made by two refractions. Alſo any two emergent rays produced will be inclined to one another in the ſame angle, as the two incident rays are inclined to one another.

8. If three homogeneous rays, making with each other equal angles, be refracted at a common point

point of incidence from a denfer into a rarer medium, the fum of the angles of refraction of the extreme rays will be greater than twice the angle of refraction of the mean ray.

Fig. 3. Let the three arches *ab*, *ag*, *ad* be in arithmetical proportion; and *bI*, *gI*, *dI* three homogeneous rays, paffing out of a denfer into a rarer medium, and refracted at *I* in the lines *IB*, *IG*, *ID*; draw the fines of incidence *bn*, *go*, *dp*, and the correfponding fines of refraction *BN*, *GO*, *DP*. Now it is known that fmall arches are as their fines, and the increments of the arches as the increments of the fines; but as the arches increafe, the increment of the arch has a greater ratio to the increment of the fine, and ftill greater as the arch is larger. Therefore the fines of refraction *DP*, *GO*, *BN* and their increments have the fame proportion amongft each other as the fines of incidence *dp*, *go*, *bn* and their increments; and fince the arches *AD*, *AG*, *AB* are larger than the arches *ad*, *ag*, *ab*, it follows, that the ratio of the arch *DG* to *GB* is greater than the ratio of *dg* to *gb*; but *dg* and *gb* are equal; therefore *DG* is greater than *GB*; and confequently the fum of the refractions *AB* and *AD* is greater than twice the refraction *AG* into the rarer medium.

9. If homogeneous rays be refracted by a prifm, the angle which the incident and emergent rays contain is then greateft, when the refractions on both fides the prifm are equal.

Fig. 4. Let the ray *PS*, parallel to the bafe of the prifm, be refracted into the lines *PM*, *SN*, making the angles

APM, *BSN* equal. Alfo let the oblique ray *PQ* be refracted into *PH* and *QL*; from *P* and *Q* erect the perpendiculars *PV*, *QV* to the fides of the prifm, and through *Q* draw *QR* parallel to the bafe of the prifm; fince the angle *QPS* = its alternate *PQR*, and *VPS* = *VQR*, the three angles of incidence *VPQ*, *VPS*, *VQP* are in arithmetical proportion; therefore the fum of the refractions of the oblique ray, out of the prifm into air, is greater than twice the refraction of the ray *PS* into the air at *P*, or than the fum of the equal refractions of the ray *PS* at *P* and *S*. And therefore if the rays *HP*, *LQ* be produced till they meet in *E*, and *MP*, *NS* till they meet in *F*, the angle *HEL* will be lefs than the angle *MFN*.

10. When diverging rays fall nearly perpendicular on a refracting plane furface, the focus after refraction is thus found: from the radiant point let fall a perpendicular on the refracting furface, and as the fine of the angle of refraction is to the fine of the angle of incidence, fo let the perpendicular diftance of the radiant from the furface be to the diftance of a point in that perpendicular, and on the fame fide, from the furface; this point will be the focus required.

11. An object being immerfed in water, appears nearer to the eye than it really is by one-fourth of its depth from the furface of the water.

12. A ray of light cannot pafs out of a denfer into

into a rarer medium, if the angle of incidence exceed a certain limit.

This limit is the angle of which the ſine is to radius, as the ſine of incidence is to the ſine of refraction out of the denſer medium into the rarer. Thus, a ray of light will not paſs out of water into air, if the angle of incidence exceed 48° 16′, the ſine of which is to radius as 3 to 4; nor out of glaſs into air, if the angle of incidence exceed 40° 11′, the ſine of which is to radius as 20 to 31; nor out of glaſs into water, if the angle exceed about 59° 20′.

13. If the angles of incidence and refraction be ſo ſmall as to be reckoned proportional to their ſines; and the angle of incidence, when light paſſes from air into glaſs, be increaſed or diminiſhed by any quantity, the refracted angle, or angle of deviation, will be increaſed or diminiſhed by one-third of that augmentation or diminution; and by one-half of the ſame, if the emergence be from glaſs into air.

14. When diverging rays fall on a denſer medium terminated by two plane and parallel ſurfaces, the focus of the refracted rays will be between the medium and the radiant point; and the diſtance between the conjugate foci will be to the thickneſs of the medium, as the difference between the ſines of incidence and refraction at the firſt ſurface to the ſine of incidence.

Fig.

Fig. 5. Let *QDE* be perpendicular to the parallel surfaces *AD*, *CE*; *QA* the incident ray, not very remote from the axis *QE*; in *DQ* produced take *R* so that $DQ : DR :: r : i$, *R* will be the focus of the refracted ray *AC*; and if *F* be taken so that $RE : FE :: i : r$, *F* will be the focus of the emergent ray *CS*; therefore since $RE : FE :: i : r$, we have $RF : RE :: i-r : i$; also $RQ : RD :: i-r : i$; therefore $QF : DE :: i-r : i$. And since *AC* approaches the perpendicular, it is evident, that *SC* produced will meet *QE* between *Q* and *D*.

When the refracting medium is glass, the interval between the conjugate foci is $\frac{1}{3}$ of the thickness of the medium.

If the thickness of the glass be a given quantity, so will the interval between the conjugate foci; and therefore if the object and the glass be given in position, the farther the eye from the object, the less will be the apparent dislocation; or if the position of the eye and glass be given, the dislocation will be less, the more remote the object.

15. Having the focus of incident rays upon a medium terminated by two plane surfaces, inclined to each other in a given angle, the focus of the emergent rays is thus found: from the radiant let fall a perpendicular on the first surface, and in it find the focus of rays refracted at that surface; from which let fall a perpendicular on the second surface, and as the perpendicular distance of this focus from the first surface is to the distance of the

he radiant from the ſame, ſo let the diſtance of he focus of rays, refracted at the firſt ſurface, from :he ſecond be to a fourth proportional, which, taken in the perpendicular to the ſecond ſurface and towards it, will give the focus required.

Fig. 6. Let $\mathcal{Q}$ be the focus of incident rays $\mathcal{Q}A$, $\mathcal{Q}B$; on the ſide of the priſm IA, produced if neceſſary, let fall the perpendicular $\mathcal{Q}G$, and in $\mathcal{Q}G$ take the point R ſo that $G\mathcal{Q} : GR :: r : i$; then will R be the geometrical focus of the pencil of refracted rays AC, BD within the priſm. Again, in the perpendicular RH to the ſide CI, produced if neceſſary, take the point F ſo that $RH : FH :: i : r$, and F will be the focus of the emergent pencil.

Becauſe $R\mathcal{Q} : RG :: RF : RH$, $\mathcal{Q}F$ is parallel to GH; therefore $\mathcal{Q}F : GH :: R\mathcal{Q} : RG$, that is, in a given ratio; hence ſince GH increaſes while GR increaſes, the interval between the conjugate foci will be greater, when the perpendicular diſtance of the radiant point from the firſt ſurface is greater; and therefore if the poſition of the eye and object be given, the apparent diſlocation of the object will be greater, when the priſm is farther from the object, or nearer to the eye.

LECTURE

LECTURE V.

1. IF parallel rays fall upon a refracting ſpherical ſurface, near the vertex, one of which paſſes through the centre, the focus is thus found: from the point of incidence of any ray draw a right line meeting the central ray, and let the length of this right line be to the ſegment of the central line intercepted between it and the centre, as the ſine of incidence to the ſine of refraction, and the interſection of this right line with the central line will be the focus required.

2. If the refracting ſurface be very ſmall, the focus is thus found: as the ſine of incidence is to the ſine of refraction, ſo is the diſtance between the focus and ſurface to the diſtance between the focus and centre.

Hence

Hence the principal focus of a convex ſpherical ſurface of glaſs is diſtant from the vertex nearly three times the radius: for in this caſe, the diſtance between the vertex and focus is to the difference between that diſtance and the interval between the centre and focus, that is, to the radius of the ſurface, as the ſine of incidence to the difference between the ſines of incidence and refraction, or as 3 to 1.

If the refraction be from glaſs into air, the diſtance of the principal focus from the ſpherical ſurface will be double the radius; becauſe the ſine of incidence is to the ſine of refraction as 2 to 3, and to their difference as 2 to 1.

The principal focus of a ſpherical convex ſurface of water is diſtant from the ſurface four times the radius; and if the refraction be made from water into air, it will be diſtant three times the radius.

3. When it is ſaid, that the focus of a ſpherical refracting ſurface is found in the manner laſt deſcribed, it is to be interpreted phyſically, not mathematically; for no ray, in truth, paſſes through the point ſo determined, except that which coincides with the axis of the ſurface; thoſe rays which fall on the ſurface nearer to the vertex having their foci farther from it.

Fig. 7. Let *AV* be a ſpherical ſurface, *RV* one of the parallel rays paſſing through *C* the centre; if *A* be the point of incidence of any ray, falling on the convex ſurface of a denſer medium; from *A* draw *AF* meeting *RV* produced in *F*, ſo that *AF* may be to *CF*, as the ſine of incidence to the

ſine of refraction, and F will be its focus. Let RB be another ray, more remote from the vertex V: this ray cannot be refracted to F, becauſe BF is leſs than AF, 3 El. 7. and therefore the ratio of BF to CF is leſs than the ratio of AF to CF, whereas the refracted ray is always to the diſtance of its focus from the centre in the ſame ratio; neither can it be refracted to any point G beyond F, becauſe the angle RBF, and therefore BFG is obtuſe; and conſequently BG greater than BF; but BF and FG are greater than BG; take $Bm = BF$, then FG is greater than mG; therefore the ratio of BG to GC is leſs than the ratio of BF to FC, that is, a fortiori, than the ratio of AF to FC; therefore the refracted ray muſt meet the axis ſomewhere between F and C, as at Q; that is, the farther the point of incidence from the vertex, the nearer is the focus to the vertex of the ſurface.

Fig. 8. If RB fall on the concave ſurface of a rarer medium, and F be the focus of RA, RB cannot be refracted to F, becauſe BF is greater than AF; therefore BF is to CF in a greater ratio than AF to CF: neither can it be refracted to any point G farther from V than F: for take $Bm = BF$, and from B erect the perpendicular Bc meeting CG in c; then $BG : Bm$ or $BF :: cG : cF$, that is, in a greater ratio than CG to CF; therefore BG is to CG in a greater ratio than BF to CF, that is, than AF to CF.

Hence the foci of the different rays occupy the ſurface of a little circle whoſe plane is perpendicular to the axis of the ſurface; and this circle is called the circle of aberration.

4. If diverging or converging rays fall on a ſmall portion of a refracting ſpherical ſurface, their focus may

may be thus found: find the focus of parallel rays coming in the contrary direction, and the distance between the conjugate foci will be a fourth proportional to the distances of the radiant from the principal focus, from the surface, and from the centre.

Hence the two foci coincide at the centre and at the surface; and the four distances, in the proposition, lie all the same way from the focus of incident rays, or two on each side.

5. If parallel rays fall nearly perpendicularly on a sphere, the focus after refraction at both surfaces will bisect the distance of the focus of refraction at the first surface from the sphere.

If the sphere be water, the focus lies at the distance of a semidiameter from the sphere; if glass, at half that distance.

6. If the line joining the centres of the surfaces of a lens be divided in the ratio of the respective radii, all the rays passing through that point will have their incident and emergent parts parallel.

A lens is a transparent body of different density from the ambient medium, and terminated by two surfaces, which are both spherical, or one plane and the other spherical.

Lenses are distinguished by the nature of their surfaces, and are of five kinds, double convex, double concave, plano-convex, plano-concave, and meniscus, which is concave on one side and convex on the other.

The point determined in this article is called the centre of the lens. Hence if the thickness of the lens be inconsider-

able, the paffage of every ray through the centre may be confidered as rectilineal.

The axis of a lens is a right line perpendicular to both furfaces.

The vertex is the point where the axis cuts the furfaces.

7. The principal focus of a lens is thus found: find the principal focus of the firft furface, this is a focus of incident rays on the fecond furface; find the principal focus of the fecond furface of rays coming the contrary way; then will the diftance of the focus of incident rays on the fecond furface from the focus required, be a fourth proportional to its diftances from the principal focus of that furface, its centre, and the furface or lens.

8. If the fine of incidence out of air into a lens be to the fine of refraction as I to R, and the radii of the two furfaces be m and n, the diftance d of the principal focus from the lens will be $=$ $\frac{mn}{m+n} \times \frac{R}{I-R}$.

If the lens be glafs, I will be to R as 3 to 2, and $d = \frac{2mn}{m+n}$.

If one radius n become infinite, or the lens become plano-convex or plano-concave, $d = 2\,m$.

If one radius n become negative, the lens will be a menifcus, and $d = \frac{2\,mn}{m-n}$.

9. Half the linear aperture of any very thin lens, of long focus, is a mean proportional between its focal

focal length, and the difference between the thickeſt and thinneſt parts of the lens nearly.

Fig. 9. Firſt, in a plano-convex and plano-concave, let the axis *Rr* be twice the radius of curvature; then will *r* be one of the principal foci of the plano-convex lens; and *R* the focus of the plano-concave; but *RS* in the former, and *rs* in the latter = the difference between the thickeſt and thinneſt parts of the lenſes; and theſe verſed ſines, when indefinely little, are third proportionals to the diameter *Rr*, and *PS* or *ps* half the linear aperture. 2. A double convex may be conſidered as conſiſting of two plano-convex lenſes; a double concave of two plano-concaves; and a meniſcus of the difference between two plano-convex lenſes; therefore if *S* be the ſemi-aperture, *D* the difference between the thickeſt and thinneſt parts of one plano-convex or plano-concave, and *d* the difference in the other, then $D + d =$ the difference between the thickeſt and thinneſt parts of the given lens $= \frac{S^2}{2m} \pm \frac{S^2}{2n} = \frac{m \pm n}{2mn} \times S^2$; and $\overline{D \pm d} \times \frac{2mn}{m \pm n} = S^2$.

In a plano-convex, double convex, and meniſcus, the difference between the thickeſt and thinneſt parts of the lens is equal to its thickneſs; therefore in theſe lenſes, half the linear aperture is a mean proportional between the thickneſs and focal length. But this propoſition is not to be relied on in lenſes of ſhort focus.

LECTURE

LECTURE VI.

1. IF rays diverging from or converging to a point in the axis of a lens fall on the lens, their focus is thus determined: from the centre of the lens take, in the axis of the pencil, the principal focal length, towards the given focus if the lens be convex, on the contrary ſide of the lens if it be concave; from the given focus, towards the principal focus, take a point whoſe diſtance from the given focus ſhall be a third proportional to the diſtances of the principal focus and centre of the lens from the ſame; that point will be the focus of the refracted rays.

The principal focus and the focus of refracted rays lie the ſame way from the focus of incident rays. And if the focus of incident rays be moved, the focus of refracted rays will move in the ſame direction; and they will meet at the ſurface.

2. In

2. In a double convex lens, the diſtance between the radiant and principal focus is to the principal focal length, as the diſtance of the object from the lens to the diſtance of the image from the ſame.

Hence if d be the diſtance of the object from an equally double convex lens, r the radius of curvature or principal focal length, and f the diſtance of the image from the lens, then $d - r : r :: d : f = \frac{dr}{d-r}$.

In an equally double concave, ſince the principal focus is to be taken on the contrary ſide of the lens, $d + r : -r :: d : f = \frac{-dr}{d+r}$.

In a plano-convex, the focal length is $2r$, therefore $d - 2r : 2r :: d : f = \frac{2dr}{d - 2r}$.

In a plano-concave $d + 2r : -2r :: d : f = \frac{-2dr}{d + 2r}$.

3. If the diſtance of an object from a double convex lens be equal to twice the focal length, the diſtances of the object and image from the lens will be equal; and their diſtance from each other will be a minimum.

The product of the difference between the diſtances of the object and image from the lens, and its principal focal length is equal to the ſquare of the focal length, which is a conſtant quantity; but if the rectangle under any two lines

lines be always of the ſame magnitude, their ſum is leaſt when they are equal.

4. If an object be ſituated at any given diſtance from a ſcreen, on which its image is to be repreſented by refraction of the rays through a double convex lens, and if the lens be moved gradually from the object to the ſcreen, it will either paſs through one or two poſitions in which the image will be diſtinctly delineated on the ſcreen, or there will be no poſition in which it will appear diſtinct.

For ſince the diſtance between the radiant and the image is a minimum, when it is equal to four times the focal length, it follows, that when it is greater than four times the focal length, there will be two poſitions of the lens, viz. one on either ſide of the point which biſects the interval between the conjugate foci, in which that interval will continue the ſame; if that interval be equal to four times the focal length, there will be but one poſition, viz. when the lens biſects the interval; and if the interval be leſs than four times the focal length, it is evident there is no poſition in which the image can be delineated diſtinctly on the ſcreen.

5. The linear magnitudes of an object and its image are to each other directly as their diſtances from the lens.

Hence in a double convex or plano-convex lens, the image may be greater, equal to, or leſs than the object; but

but in a concave lens, the magnitude of the image will always be leſs than the object.

6. If the eye be cloſe to a concave lens, the apparent magnitude of the image of any object ſeen through it will be equal to that of the object; as the eye gradually recedes from the lens, the image will appear to diminiſh, and the limit of the ratio of the apparent magnitude of the image and object will be the ratio of their actual magnitudes.

By the apparent magnitude of the image or object is underſtood the angle which they ſubtend at the eye, and not the apparent magnitude according to the judgment which the mind forms of them. Their apparent magnitude therefore depends on two circumſtances, their actual magnitude, and their diſtance from the eye; and ſince the image is ſmall when compared with its diſtance from the eye, it will ſubtend an angle at the eye, which is as the length of the image directly, and the diſtance of the eye from it inverſely.

7. If the diſtance of an object from a double convex lens be leſs than the focal length, as the eye recedes gradually from the lens, the ratio of the apparent magnitude of the image and object will continually increaſe; and the limit of its increaſe will be the ratio of the real magnitude of the image and object.

8. If the diſtance of an object from a double convex lens be equal to the focal length, the ap-

parent magnitude of the image with respect to the object will continually increase without limit, as the eye recedes from the lens.

9. If the distance of the object from the lens be greater than the focal length, the ratio of the apparent magnitude of the image and object will increase without limit, according as the eye recedes from the lens towards the image.

10. If the eye be farther from the lens than the image, and the ratio of its distance from the image to its distance from the object be less than the ratio of the magnitude of the image to that of the object, the image will appear greater than the object; if the ratio of these distances be equal to the ratio of the magnitudes, the object and image will appear equal; and if it be greater, the image will appear less.

11. If the object be infinitely distant, it will appear magnified, of its usual size, or diminished, according as the distance of the eye from the lens is less than, equal to, or greater than twice the focal length.

12. Images formed by a concave lens, or by a convex lens when the object is nearer to it than its focal length, are erect and imaginary.

13. The image of an object whose distance from a convex

a convex lens is greater than the focal length, is inverted and real.

14. The image ſeen through a concave lens is always darker than the object.

15. If the object be in the principal focus of a convex lens or within it, the image will appear brighter than the object.

16. If the object be beyond the principal focus, the image ſeen through the lens will appear leſs bright than the object, equally bright with it, or brighter than it, according as its diſtance from the lens is leſs than twice the focal length, equal to, or greater than it.

17. The brightneſs of the image of any luminous object formed by a convex lens, will be as the area of the lens directly, and inverſely as the ſquare of the diſtance of the image.

18. The degrees of heat in the foci of different convex lenſes expoſed to the ſun's rays, are as their areas directly, and inverſely as the ſquares of their focal lengths. And the heat in the focus of a lens is to the ſun's direct heat, as the area of the glaſs to the area of the image.

LECTURE VII.

1. IF the rays of the ſun, refracted by a triangular glaſs priſm, were homogeneous, there is a certain poſition of the priſm in which the image would appear orbicular.

Fig. 10. Let XL, YK be two rays, the one coming from the upper limb of the ſun, the other from the lower, and croſſing each other in F, the aperture in the window ſhutter EG; let the priſm ABC be turned round its axis ſo, that the refraction of the ray YK, at its incidence at K, may be equal to the refraction of the ray XL at its emergence at I; then ſuppoſing the ray TI to go backwards, the refraction of the ray XL at L will be equal to the refraction of YK at H; therefore the ſum of the refractions of the ray YK, at K and H, will be equal to the ſum of the refractions of the ray XL at L and I; and therefore

therefore the two rays, being equally refracted, have the ſame inclination to one another after refraction as before, that is, half a degree, equal to the ſun's appparent ſemi-diameter. So then *PT* would ſubtend an angle at the priſm of half a degree, and therefore would be equal to the breadth of the image; and conſequently the image would be circular.

2. The ſun's light conſiſts of rays of different colours, and different degrees of refrangibility.

If the ſun's rays be admitted into a dark room through a ſmall hole in a window ſhutter, and be refracted through a glaſs priſm, the image will be an oblong figure terminated with ſtraight parallel ſides and ſemicircular ends, the length of which is about five times its breath. The whole image conſiſts of ſeven diſtinct colours, lying in the following order, red, orange, yellow, green, blue, indigo, and violet, and are called the Priſmatic colours from the experiment.

The ſeparated rays deviate unequally from the courſe of the incident ray, whence they are ſaid to be differently refrangible. The refrangibility of the rays anſwers to the order in which the colours appear in the priſmatic ſpectrum, the red being the leaſt, and the violet rays the moſt refracted. Any of the ſeven colours ſeparated from the reſt is called an homogeneal ray.

The angle contained between the ſides of the priſm through which a ray paſſes, is called the refracting angle.

It is always ſuppoſed in optical experiments, that when a ray is refracted through a priſm, the incident and emer-

gent

gent rays are in a plane which is perpendicular to the axis of the prism.

3. If the prism be turned about its axis, the spectrum will first ascend and then descend, or the contrary; in the limit between which motions, when the image is stationary, the refractions on both sides are equal.

In this position of the prism all experiments on heterogeneal light are supposed to be made, unless some other position is described: 1. Because the refractions of the rays may be thus measured without any other instrument than a quadrant. 2. The refraction by being doubled can be more certainly measured. 3. The prism may be placed in this position with the greatest facility. 4. A given error in the position of the prism will produce a much less effect.

4. If the prism be turned about its axis so as that the rays shall be incident more obliquely on the second surface, the image will become longer; if it be turned in the contrary direction, so as that they shall be incident more obliquely on the first surface, it will become shorter.

Fig. 11. Let the solar beam *SL* be refracted by the prism *ABC*, forming the spectrum *PVT*, the rays *LK* of mean refrangibility, within the prism, being parallel to the base; if the prism be turned on its axis according to the order of the letters *ACB*, so that the rays shall be incident more obliquely on the first surface, the violet ray *LHP* will approach to parallelism with the base, and therefore the angle which

which the violet incident and emergent rays contain, will increaſe, that is, the violet extremity P of the ſpectrum will deſcend. At the ſame time the red rays will recede farther from the poſition, in which their parts within the priſm are parallel to the baſe; therefore the angle contained by the incident and emergent red rays will decreaſe; that is, the red extremity of the ſpectrum will aſcend, and thus the length of the ſpectrum will be contracted. And the contrary will happen, if the priſm be turned on its axis in the contrary direction.

5. The oblongation of the ſpectrum does not ariſe from any ſplitting of the rays, nor from any inequality of incidence on the priſm.

6. If the common ſine of incidence from crown glaſs into air be 50, the ſines of refraction of the leaſt and moſt refrangible rays will be 77 and 78 reſpectively.

Hence in ſmall refractions, where the angles and ſines are nearly proportional, the refracted angles, that is, the refractions of the leaſt and moſt refrangible rays will be as 27 and 28; and the difference of the refractions of the leaſt and moſt refrangible rays will be about $\frac{1}{27\frac{1}{2}}$ part of the whole refraction of the mean refrangible rays.

7. If heterogeneous rays diverging from a lucid point be refracted acroſs a priſm, given in poſition, to the eye, the ſum of the angles formed by theſe rays at the eye and lucid point will be a given quantity,

tity, the mean refractions on both sides the prism being equal.

Fig. 12. Let the heterogeneous rays FD, FE be refracted across the prism ACB to the eye at X: let two heterogeneous rays, similar to FD, FE as to refrangibility, be incident on the prism in the direction LM parallel to the right line bisecting the angle DFE; the deviation of FD is nearly equal to the deviation of LMO, and the deviation of FE to that of LMN; therefore the difference of the deviations of FD and FE is equal to the difference of the deviations of LMO and LMN, that is, to the angle NMO which the incident and emergent rays contain. But the deviation of $FD =$ the angle $PVX = VFX + VXF$, and the deviation of $FE = QRX = RFX + RXF$; therefore the difference of the deviations $= DFE + GXH = NMO$, a given quantity, because the rays within the prism are supposed parallel to the base. Lect. Opt. P. 1. Prop. 27.

8. If heterogeneous rays diverging from a given point be refracted across a prism to the eye, the divergence in entering the eye will be greater, the more remote the object, the prism and eye remaining fixed. If the eye and object be given in position, the nearer the prism is to the object, the less will be the divergence.

For since the rays DG, EH within the prism are very nearly parallel to the base, the angle GXH will be to DFE, as DF to GX; and $GXH : DFE + GXH$, a constant quantity, $:: DF : DF + GX$; therefore $GXH :: \frac{DF}{DF+GX}$, or,

or, when DF is ſmall with reſpect to GX, : : $\frac{DF}{GX}$; therefore when GX is given, the divergence is directly as DF, the diſtance of the object.

Hence the divergence is greateſt when the object is infinitely diſtant; and it vaniſhes when the object touches the priſm.

9. If parallel heterogeneous rays, paſſing through a ſmall aperture in a window ſhutter, fall obliquely on a glaſs parallelepiped, they will generate a coloured ſpectrum, in which the colours will preſerve the ſame poſitions and the ſame breadth at all diſtances from the parallelepiped; and the breadth of the ſpectrum will be proportional to the breadth of the parallelepiped.

10. If heterogeneous rays flowing from the different points of the ſun's diſc, and croſſing each other in a ſmall aperture in a window ſhutter, fall obliquely on a glaſs parallelepiped, they will generate a coloured ſpectrum; and as the diſtance from the parallelepiped increaſes, the figure of the ſpectrum will become more orbicular; the green in the middle will by degrees migrate into whiteneſs, and the whiteneſs expand; and the extreme colours will continue, but gradually grow more languid by the dilatation of the image.

Fig. 13. If the heterogeneous beams XH, YH, SH flowing from the upper and lower limbs and centre of the ſun,

crofs each other in the aperture F, and falling on the parallelepiped be refracted by it, Pp the axis of the emergent cone of purple rays will be parallel to SH the axis of the cone of incident rays; in the like manner, Tt the axis of the emergent cone of red rays will be parallel to SH; therefore as the centres of the circles p, t always preferve the fame diftance from each other, and the circles themfelves enlarge as the diftance is increafed, the colours will gradually intermix, and become more dull and languid, and the figure of the fpectrum will become more orbicular. At O where the extreme cones interfere, whitenefs will be generated in the middle of the fpectrum.

11. If heterogeneous rays diverging from a lucid point, and falling obliquely on a glafs parallelepiped, be refracted acrofs it to the eye, the divergence of the rays in entering the eye will diminifh, according as the object, or eye, or both recede from the parallelopiped.

Fig. 14. Let X be the lucid point emitting heterogeneous rays to the eye at S, acrofs the parallelepiped $ABCD$, thefe rays will have their incident and emergent parts XM, pS, and XN, tS parallel to each other, and therefore the triangles pSt, MXN will be fimilar, the purple rays, upon account of their greater refrangibility, diverging more on either fide at M and p from their rectilineal courfe than the red rays; whence the heterogeneous rays muft neceffarily crofs each other fomewhere, as at O, within the prifm, making the triangles MON, pOt alfo fimilar; and therefore the points S, O, X lie in directum. Let the right line SX

meet

meet the parallelepiped in K and L, then $SK : LX :: SO : OX$, and $SK + LX : LX :: SX : OX$; if now SK be increaſed, LX remaining unvaried, the ratio of $SK + LX$ to SX, and therefore, alſo the ratio of LX to OX will be increaſed, that is, the interſection O will approach towards X; and therefore the heterogeneous rays, which are refracted to O, will fall leſs obliquely on the parallelepiped, that is, the angle MXN will diminiſh, and therefore alſo the angle pSt which is equal to it. And the like will happen, if SK remaining unvaried, LX be increaſed.

12. The ſun's rays are differently reflexible, and thoſe rays which are the moſt refrangible, are alſo the moſt reflexible.

The ſine of incidence is to the ſine of refraction in the red rays, paſſing out of glaſs into air, as 50 : 77; wherefore they will be reflected from the ſurface of the glaſs, if the angle of incidence exceed 40° 29′, the ſine of which angle is to radius as 50 : 77; the ſine of incidence is to the ſine of refraction of the violet rays as 50 : 78, theſe rays therefore will be reflected, if the angle of incidence exceed 39° 52′.

13. Candle light is of the ſame nature as the light of the ſun.

For rays from a candle may be ſeparated into the ſame colours with thoſe in the ſolar ſpectrum, and they lie in the ſame order; but their proportions are different.

LECTURE VIII.

1. WHEN a ray of light emerges obliquely from one medium into another, which refracts from the perpendicular, the greater is the difference of their refractive denfity, the lefs obliquity of incidence is requifite to caufe a total reflection, and the ftronger the reflection will be.

2. The refraction of light is probably produced by the attractive force of the refracting medium.

Becaufe fuch a force is not only fupported by the analogy of nature, but is likewife adequate to the effects thus attributed to it.

3. The augmentation or diminution of the velocity of the refracted rays, caufed by the attraction of the refracting medium, is not varied by any variation of the angle of incidence.

Hence it follows, that the fines of incidence and refraction are to each other in a conftant ratio.

Hence alfo a ray paffing obliquely from a rarer into a denfer

denser medium will approach towards the perpendicular to the refracting surface at the point of incidence; and will recede from that perpendicular in passing from a denser to a rarer.

It is to be observed, that a repulsive force in the refracting mediums will solve the phænomena of refraction equally well with an attractive force.

4. The different refrangibility of the rays of light does not depend either on the different magnitude of the particles, or the different velocity of the rays.

The different refrangibility of the rays probably depends on the different degrees of elective attraction, which subsists between the refracting body and the different sorts of rays.

5. Bodies reflect and refract light by one and the same power, variously exercised in various circumstances.

This appears by several considerations: 1. When light goes out of glass into air, as obliquely as it can, if the incidence be increased, it will be totally reflected. 2. Light is alternately reflected and transmitted by thin plates of glass for many successions, according as the thickness of the plate increases in arithmetical progression. 3. Those surfaces of transparent bodies which have the greatest refracting power, reflect the greatest quantity of light.

6. The reflection of light is not caused by its actually impinging upon the solid parts of the reflecting body, but by some power diffused over the surface, and which acts in a direction perpendicular to it.

For

For when a ray passes obliquely from a denser to a rarer medium, there is a certain angle of incidence, at which a total reflection begins; and the greater the difference between the refractive powers of the mediums, the less the angle of incidence when this total reflection takes place, so that the force, whether attractive or repulsive, which permits the ray to be refracted at a certain incidence, will be so far increased by a greater obliquity, as to produce reflection. Prin. Mathem. Lib. 1. Prop. 96.

Now if we suppose that the attractive force of the medium, through which a ray passes, is indefinitely greater than that which prevails at the surface of the body on which it falls, the rays will be reflected at all angles of incidence: and in the same manner as refraction is not made in the point of incidence, but gradually by a continual *inflection* of the rays; so neither will reflection be performed in the point of incidence, but the ray will describe an indefinitely little parabolic arc, whose axis will bisect the angle made by the incident and reflected rays, and will be perpendicular to the reflecting surface.

7. The forces of bodies to refract and reflect light are as the squares of the co-tangents of the greatest refraction.

The squares of the co-tangents of greatest refraction, and by consequence the refractive forces of bodies, are found to be nearly as their densities, except that unctuous and sulphureous bodies refract more than others of the same density.

LECTURE

LECTURE IX.

1. INFLECTION is that change in the direction of a ray of light, which it undergoes in paſſing by the edges of bodies, and very near them.

2. If the ſun's rays be admitted through a ſmall hole into a darkened chamber, and a thin plate two inches long and about half a line broad be perpendicularly expoſed to the rays, a faint light will be ſeen diſperſed over the ſhadow, when received perpendicularly on a ſmooth white ſurface; the ſhadow alſo will be broader, than if the light had all proceeded from the hole in right lines, and will be terminated by various fringes of colours, the moſt refrangible rays, in each fringe, being neareſt the body.

From

From this experiment it appears, that ſome of the rays which paſs near the edges of the plate, are bent from the body, which is ſimilar to reflection; and others are bent towards the body, into the ſhadow, which is ſimilar to refraction.

3. The coloured fringes which border the ſhadows of bodies do not ariſe from any new modifications impreſſed on the rays of light, but ſolely from their various inflections, by which they are ſeparated from each other.

It has been attempted to ſolve the phænomena of inflection by the hypotheſis of an atmoſphere ſurrounding the inflecting body, which, variouſly refracting and reflecting the rays, produces theſe extraordinary appearances; but all ſuch attempts hitherto made appear unſatisfactory.

As inflection reſembles both reflection and refraction, and the inflecting power alſo acts on ſome rays more than on others, by which they are ſeparated from each other, Newton concludes, that the power by which the rays of light are inflected, is one and the ſame with that by which they are refracted and reflected, though the particular manner in which the phænomena are produced, is yet unknown.

LECTURE X.

1. AS the rays of the fun differ in refrangibility, fo do they alfo in their difpofition to exhibit different colours; and to the fame degree of refrangibility, in a given medium, always belongs the fame colour; and to the fame colour, the fame degree of refrangibility.

2. That fpecies of colour which is peculiar to any kind of rays, is not changeable by refraction or reflection.

3. Bodies appear of particular colours by their difpofition to reflect the rays of one colour more copioufly than another.

Hence if the fun's light confifted but of one fort of rays, all things would appear of the fame colour.

Bodies appear of different colours in candle-light and day-light, becaufe the component colours of each are to each other in different proportions.

4. Tranſparent bodies appear of different colours, by tranſmitting ſome rays more copiouſly than others, and ſtifling thoſe which they do not tranſmit.

If the light which tranſparent coloured bodies tranſmit be intercepted, and the other rays fall on them, theſe bodies do not vary from their former colour, but become entirely black.

5. The diſpoſition of bodies to reflect or tranſmit ſome rays more copiouſly than others, depends on the denſity or magnitude of their component particles.

6. Colours are either original or compounded; the original or primary colours are thoſe *which appear* in the ſolar ſpectrum; the compounded are thoſe which reſult from the union of primary colours; and which differ from the primary only in this, that they can be decompounded; whereas the primary or priſmatic colours cannot.

7. All the primary colours mixed in that proportion which they obtain in the ſolar ſpectrum, conſtitute whiteneſs.

8. The blackneſs of bodies proceeds from their incapacity to reflect any of the rays of light.

LECTURE

LECTURE XI.

1. THE rain-bow is formed by the ſolar rays entering the drops of falling rain, and being there refracted to the farther ſurface, emerging from the drops after one or more reflections; and at their emergence, as well as at their entrance ſuffering a refraction by which the compound rays are ſeparated, they enter the eye of a ſpectator, and exhibit the priſmatic colours.

The primary bow is that which is moſt uſually ſeen in the heavens, and is formed by two refractions, and one intervening reflection of the rays. The ſecondary bow is formed by the rays emerging after two refractions, and two intervening reflections.

2. Of the ſolar rays which fall parallel and contiguous upon a ſpherical drop of water, thoſe

which are effective, or can produce a rain-bow, must also emerge parallel and contiguous.

Because otherwise they will not enter the eye of the spectator in sufficient abundance, and therefore with sufficient strength to excite a distinct sensation; diverging rays would also by their various crossings and intersections dilute the colours.

3. The effective rays which emerge after one reflection, have all one common point of reflection, on the opposite surface of the drop.

4. The effective rays which emerge after two reflections, have their reflected parts parallel to each other within the drop, between the two points of reflection.

5. The effective rays which emerge after one reflection, have their angle of incidence so ordered, that its nascent increment or smallest increase is double the contemporary increase of the angle of refraction.

Fig. 15. Let *RI*, *ri* be the incident, and *EM*, *em* the emergent efficacious rays, having one common point of reflection *Z* on the opposite surface of the drop; the nascent angle *ICi* is the nascent increment of the angle of incidence; and *i ZI* is the contemporary increment of the angle of refraction; but the former angle, being at the centre, is double the latter at the periphery, insisting on the same arch.

6. The nascent increment of the angle of incidence

dence of the effective rays which emerge after two reflections, is triple the contemporary increment of the angle of refraction. And in general, the nascent increment of the angle of incidence of the effective rays which emerge after any number of reflections n, is to the contemporary increment of the angle of refraction, as $n+1$ to 1.

Fig. 16. For Ii is the nascent increment of the angle of incidence, as before, and since CZY, Czy are the angles of refraction, the angle ZCz, measured by the arch Zz, is the increment of the angle of refraction, because ZY is parallel to zy. But 2 Zz (= arch ZY — arch zy = IZ — arch iz) = Ii — Zz; therefore Ii = 3 Zz; and so on.

7. The tangents of the angles of incidence and refraction of the effective rays are to each other as 2 to 1, when they emerge after one reflection; and as 3 to 1, when they emerge after two reflections: and, in general, as $n+1$ to 1 after n number of reflections.

Fig. 17. For in two angles whose sines are in a given ratio, their contemporary increments are as the tangents of those angles. In the circle CAB let de be the nascent increment of the arch Be, and di the contemporary increment of the sine; from similar triangles, de : di : : Ce (radius) : Cf, that is, the nascent increment of the arch is as the nascent increment of the sine directly, and as the cosine inversely; but since the sines are in a constant ratio, so are their increments; therefore the nascent increment of the arch

arch is directly as the sine and inversely as the cosine, that is, directly as the tangent.

8. There is a certain point on every drop, on which rays falling will emerge parallel after one reflection; and another, on which if they fall, they will emerge parallel after two reflections.

In the beginning of the quadrant the tangents are very nearly as the sines of the angles of incidence and refraction, that is, as 4 to 3, which is a less ratio than 2 to 1 or 3 to 1; but, in the end of the quadrant, the tangent of incidence becomes infinite, while the tangent of refraction is a finite quantity; therefore there must be some intermediate state, in which the tangents of incidence and refraction are as 2 to 1, 3 to 1, or $n+1$ to 1.

To find the position of the point, all that is necessary is to find two angles whose sines are in the ratio of the sines of incidence and refraction of every particular coloured ray, and whose tangents in the first bow are as 2 to 1, in the second as 3 to 1, and in general as $n+1$ to 1, n denoting the number of reflections; which is evidently a problem, that admits of a solution; because there are two unknown quantities, and two equations likewise.

9. The cosine of incidence of the effective rays which emerge after any number of reflections n, is to radius as $\sqrt{I^2 - R^2}$ to $\sqrt{\overline{n+1}^2 - 1\, R^2}$; I denoting the sine of incidence, and R the sine of refraction.

Let x and $y =$ the cosines of incidence and refaction respectively;

ſpectively; then the ſquares of their ſines will be $1-x^2$, and $1-y^2$, radius being unity; and the ſquares of their tangents will be $\frac{1-x^2}{x^2}$, and $\frac{1-y^2}{y^2}$; wherefore, by the conditions of the problem, $1-x^2 : 1-y^2 :: I^2 : R^2$, and $\frac{1-y^2}{y^2} : \frac{1-x^2}{x^2} :: 1 : \overline{n+1}^2$; and joining theſe ratios, $x^2 : y^2 :: I^2 : \overline{n+1}^2 R^2$, and $y^2 = \frac{\overline{n+1}^2 R^2 x^2}{I^2}$; wherefore ſubſtituting this value in the firſt equation, we have $1-x^2 : \frac{I^2-\overline{n+1}^2 R^2 x^2}{I^2} :: I^2 : R^2$, therefore $1-x^2 = \frac{I^2-\overline{n+1}^2 R^2 x^2}{R^2}$; whence $x = \frac{\sqrt{I^2-R^2}}{\sqrt{\overline{n+1}^2-1}\, R}$, the coſine of incidence. But $y : x :: \overline{n+1} R : I$, and therefore the coſine of refraction is alſo known.

If $n=1$, $\sqrt{\overline{n+1}^2-1}=\sqrt{3}$; if $n=2$, $\sqrt{\overline{n+1}^2-1} = \sqrt{8}$; if $n=3$, $\sqrt{\overline{n+1}^2-1}=\sqrt{15}$, &c. the numbers 3, 8, 15, &c. being gathered by the continual addition of the terms of the arithmetical ſeries 3, 5, 7, 9, &c.

10. The angle which the incident and emergent rays contain after one reflection is equal to the difference between four times the angle of refraction and twice the angle of incidence; and after two reflections it is equal to the difference between ſix times the angle of refraction, and twice the angle of incidence; and ſo on, always adding twice the angle of refraction for each reflection.

Fig.

Fig. 18. Let $CIZE$ be a ſection through the centre of a drop, in the plane of which let RI be an incident ray, which after two refractions in the points I, E, of incidence and emergence, and one intervening reflection in Z, emerges in the line EM. Let EM produced meet the incident ray RI, produced alſo, in X, and from the centre C let the ſemi-diameters CI, CZ be drawn. Becauſe the angles CZI, CZE, and alſo the angles ZIX, ZEX are equal, CZ produced will paſs through X, and biſect the angle IXE. Now in the two triangles CIX, CIZ, becauſe the vertical angle at C is a right one, $IXZ + CIX = CIZ + CZI = 2\ CIZ$; therefore $IXZ = 2\ CIZ - CIX$, and $2\ IXZ$ or $IXE = 4\ CIZ - 2\ CIX$, that is, the angle which the incident and emergent rays contain = 4 angle of refraction — 2 angle of incidence.

Again, let the ray RI, after two reflections in Z and E, emerge in the line FR, meeting RI and XE in R and M; becauſe the refractions in F and E are equal, the angles XEZ and EFM are equal, and $FME + MFE = FEX = FEZ + XEZ$, therefore $FME = FEZ = 2$ angle of refraction; but the angle NRM contained by the incident and emergent rays $= IXE + FME = 4$ angle of refraction — 2 angle of incidence + 2 angle of refraction = 6 angle of refraction — 2 angle of incidence. And ſo on, always adding twice the angle of refraction, for every incidence, to the former angle contained by the incident and emergent rays.

11. In the primary bow, the leaſt refrangible rays, and in the ſecondary bow the moſt refrangible are outermoſt.

Since

Since the ratio of the sines of incidence and refraction are different in all the rays of different colours, it is evident that the efficacious incident and emergent rays contain different angles; and since the deviation of the red rays is less than that of any other, it follows, that the incident and emergent red rays after one reflection contain a greater, and after two reflections a less angle than any other rays: and consequently the red rays will be exterior in the primary bow, and interior in the secondary bow.

The efficacious incident and emergent red rays form an angle of 42° 2′, and the violet 40° 17′ after one reflection. And when they emerge after two reflections, the red efficacious incident and emergent rays contain an angle of 50° 57′, and the violet 54° 7′; and the intermediate coloured rays intermediate angles.

12. The eye is in the vertex of a cone, in the surface of which all the drops lie which render the rays of any one colour efficacious.

Hence the bow appears circular. The axis of the cone is directed to the sun, and hence the sun is directly opposite to the centre of the bow.

13. The apparent semi-diameter of the bow is equal to the angle contained by the incident and emergent rays.

14. The apparent semi-diameter of the bow is equal to the apparent altitude of the highest point of the bow together with the altitude of the sun's centre above the horizon.

Hence if the sun and spectator's eye be in the horizon, the bow will appear an exact semi-circle; and the visible segment above the horizon will continually diminish as the altitude of the sun increases, until at length, when that altitude becomes equal to 42° 2′, the primary bow will be invisible; and for the same reason, no secondary bow can be observed, unless the altitude of the sun's centre above the horizon be less than 54° 7′

LECTURE

LECTURE XII.

1. If homogeneal rays iſſue from an object placed in the axis of a convex lens, the extreme image, or that which is formed by the rays tranſmitted through the extremity of the aperture, will be nearer to the lens than the principal image, or that which is repreſented by the rays neareſt to the axis.

2. If parallel homogeneal rays fall perpendicularly on the plane ſide of a plano-convex lens, the longitudinal aberration will be to the verſed ſine of the arch intercepted between the point of incidence and the axis, as the ſquare of the ſine of refraction to the rectangle under the ſine of incidence and the difference of the ſines very nearly.

Fig. 19. When the refraction is made from a denser to a rarer medium, the focus F of the refracted ray, in the axis CF, lies between the surface and the principal focus P; with the centre F and radius FA describe the arch AK, and let fall the perpendicular AG. Then CF is to AF or KF, as the sine of refraction to the sine of incidence, or as r to i; that is, for the same reason, as CP to VP; therefore, $(CF - CP)\ PF : (KF - VP)\ KV - PF :: CP : VP :: r : i$; and $PF : KV :: r : r - i$. But $KG : GV :: CV : FK$; whence conjointly $KV : GV :: (CV + FK)\ CF : FK :: r : i$; then by compounding this and the foregoing proportion, we have $PF : GV :: r^2 : \overline{r - i} \times i$.

3. If $i : r$ as the sine of incidence to the sine of refraction out of the denser into the rarer medium, S the sine of half the aperture of the lens, R the radius of curvature, and d the principal focal length, then the longitudinal aberration will be equal to $\frac{r^2}{\overline{r-i} \times i} \times \frac{S^2}{2R}$, or $\frac{r^2}{\overline{r-i}|^2} \times \frac{S^2}{2d}$.

For the longitudinal aberration $= \frac{r^2}{\overline{r-i} \times i} \times GV$; but $GV = \frac{AG^2}{2 . CV}$, and $CV = \frac{r - i}{i} \times VP$; therefore the aberration $= \frac{r^2}{\overline{r - i}|^2} \times \frac{AG^2}{2VP}$.

4. The lateral aberration will be equal to $\frac{r^2}{\overline{r-i}|^2} \times \frac{S^3}{2d^2}$, or to $\frac{r^2}{i^2} \times \frac{S^3}{2R^2}$.

For

For let the refracted ray produced cut the line Px, perpendicular to the axis, in x; then the lateral aberration $Px : PF :: AG : GF$ or VP, nearly.

5. When the radius of curvature, or the focal length is given, the longitudinal aberrations are as the squares, and the lateral aberrations as the cubes of the linear apertures of the refracting surface, or of a plano-convex lens whose plane side is turned towards the incident parallel rays.

6. The diameter of the least circle of aberration is equal to one fourth of the diameter of the circle of aberration at the principal focus.

Fig. 20. Let the refracted ray NT meet the axis in T, and the extreme ray AF in S: draw SO perpendicular to VP, Px being the lateral aberration at the principal focus P. Then, supposing the ray Ax fixed, as the point of incidence N moves from the vertex V, the perpendicular SO will first increase, because the angle VTN increases, and afterwards it will decrease, because the line FT continually decreases; and when SO is greatest, it is evident, that all the rays, on the same side, will pass through it; and then the circular section of all the rays, in the point O, will be the least possible. Now, by similar triangles, $TO : SO :: TE$ or $FG : EN$ or GM; and $SO : FO :: AG : FG$; therefore, ex æquo, $TO : FO :: AG : GM$; and by compounding, $TF : FO :: AM : GM$. Again, $PF : PT :: AG^2 : GM^2$ (Art. 5.) and by division, $PF : TF :: AG^2 : AM \times BM$; but we had $TF : FO :: AM : GM$, and FO

: SO

$: SO :: FG : AG$; therefore, ex æquo and by compounding, $PF : SO :: AG \times FG : BM \times GM$. But PF, AG and FG are given quantities, therefore SO is greatest, when $BM \times GM$ is greatest, that is, when $BM = GM$. And since $TO : FO :: AG : GM :: 2 : 1$, and $PT : PF :: GM^2 : AG^2 :: 1 : 4$, it follows, that $SO : Px :: FO : FP :: 1 : 4$.

7. The aberration arising from the spherical figure of a lens is as the cube of the angle in which the extreme pencils of the field are inflected, when the focal length is given.

For when the focal length is given, the refraction is as the aperture; but the aberration is as the cube of the aperture, that is, as the cube of the angle of refraction.

8. If any number of lenses of a given aperture be so combined together, as that the whole refraction shall be equally divided between them, the sum of the aberrations produced in this compound lens will be to the aberration produced in a single lens of the same focal length, as unity to the square of the number of lenses.

Let A be the angle of refraction produced by a lens compounded of lenses whose number is n, between which the whole refraction is equally divided; the angle of refraction in each lens separately $= \frac{A}{n}$; and the aberration in each $= \frac{A^3}{n^3}$; therefore the aberration produced by the single lens is to the sum of the aberrations produced by all the combined lenses, as $A^3 : \frac{A^3}{n^3} \times n :: n^2 : 1$.

9. If

9. If parallel rays fall perpendicularly on the plane fide of a plano-convex lens, the convex fide being generated by the revolution of an hyperbola round its axis, in which the axis is to the interval between the foci, as the fine of incidence to the fine of refraction out of glafs into air; the ray will be refracted, without aberration, to the focus of the oppofite hyperbola. And if the lens be a double convex generated in like manner by two hyperbolas, rays flowing from a radiant in the focus of the oppofite hyperbola of one of them, will be refracted, without aberation, into the focus of the oppofite hyperbola of the other.

See Ham. Con. Lib. 2. Prop. 23. If the radiant be not in the axis, the rays iffuing from it will not be refracted without aberration.

10. If parallel rays be reflected at a fpherical concave, the longitudinal aberration from the geometrical focus is nearly equal to the fquare of the right fine of half the aperture, divided by four times the radius of curvature.

Fig. 21. Let RA be the incident and AF the reflected ray, and P the principal focus; with F as a centre defcribe the arch AK, and let fall the perpendicular AG; fince the verfed fines of arches are as the fquares of their right fines divided by their diameters, $GK : GV :: CV : FA :: 2 : 1$, nearly; and $GV = VK$; but $2\ PF = 2CF -$

2CP

$2CP = CK - CV = VK = GV$; therefore $PF = \frac{GV}{2}$ $= \frac{AG^2}{4CV}$, nearly.

11. The lateral aberration is equal to the cube of the ſine of half the aperture, divided by twice the ſquare of radius.

For $Px : PF :: AG : FG$ or $\frac{1}{2} CV$, nearly.

12. When the radius of curvature, or focal length is given, the longitudinal aberrations are as the ſquares, and the lateral aberrations as the cubes of the linear apertures of the ſpeculum.

13. The diameter of the leaſt circle of aberration is one-fourth of the diameter of the circle of aberration at the principal focus.

This is demonſtrated in the ſame manner as art. 6.

14. If the focal diſtances and apertures of a concave ſpeculum and a plano-convex lens be both the ſame, the diameter of the circle of aberration, cauſed by the figure, will be thirty-ſix times leſs in the ſpeculum than in the lens.

For by Art. 4 and 11. the aberrations in the lens and ſpeculum are as $\frac{r^2}{\overline{r-i}|^2} \times \frac{S^3}{2d^2} : \frac{S^3}{8d^2}$, that is, becauſe S and d are given, and $r : i :: 3 : 2$, as $36 : 1$.

15. If a convex lens of crown glaſs be expoſed to heterogeneal parallel rays, they will come to their foci

foci at different diſtances from the lens, and the focus of the moſt refrangible will be nearer to the lens than the focus of the leaſt refrangible by the $27\frac{1}{2}$ part of the diſtance of the mean refrangible rays.

The difference between the diſtances of the foci of the leaſt and moſt refrangible rays will be different in lenſes of different diſperſive powers. See Lect. 16.

16. If heterogeneal rays, flowing from a lucid point in the axis of a convex lens of crown glaſs, be made to converge to points not too remote from the lens, the focus of the moſt refrangible will be nearer to the lens than the focus of the leaſt refrangible, by a diſtance, which is to the $27\frac{1}{2}$ part of the diſtance of the mean refrangible rays, as the diſtance between that focus and the lucid point, is to the diſtance between the lucid point and the lens.

Fig. 22. Let R be the focus of incident rays on the lens C, Q the conjugate focus of the moſt refrangible, and F of the leaſt refrangible rays, T the principal focus of the moſt, and P of the leaſt refrangible rays, coming in the contrary direction; then $RP : RC :: RC : RF$, and $RC : RT :: RQ : RC$, therefore $RP : RT :: RQ : RF$, and $PT : RT :: QF : RF$; but $PT = \frac{1}{27} TC$, therefore $QF : \frac{1}{27} TC :: RF : RT$; alſo $TC : QC :: RT : RC$, therefore $QF : \frac{1}{27} QC :: RF : RC$.

17. The extreme image of an object formed by

a convex lens is coloured and indiſtinct, if the proportion of the diameter of the aperture to the principal focal length exceed a certain limit.

18. The diameter of the leaſt circular ſpace into which a convex lens of crown glaſs can collect all ſorts of parallel rays, is about the $\frac{1}{55}$ part of the whole linear aperture.

19. The angle of aberration varies as the diameter of the aperture directly, and the focal length inverſely.

20. The diameter of the circle of aberration, cauſed by the ſphericalneſs of the figure of a lens, is inconſiderable, when compared with that which ariſes from the unequal refrangibility of the rays.

The lateral aberration, at the principal focus, ariſing from the ſphericalneſs of the figure $= \frac{r^2}{i^2} \times \frac{S3}{2R^2}$; but the diameter of the leaſt circle of aberration is half this quantity, and therefore $= \frac{r^2}{i^2} \times \frac{S3}{4R^2}$; let r be to $i :: 31 : 20$, $S = 2$ inches, $R = 600$ inches, then the diameter of the leaſt circle of aberration $= \frac{31^2 \times 8}{20^2 \times 4 \times 600^2} = \frac{961}{72000000}$ of an inch. But the diameter of the leaſt circle of aberration, ariſing from the unequal refrangibility is $= \frac{4}{55}$ of an inch; hence the former aberration is to the latter as 1 to 5449.

21. The

21. The density of the rays in any point of the circle of aberration is directly as its distance from the circumference, and inversely as its distance from the centre.

Fig. 23. For supposing the heterogeneal rays that are refracted at the points *A*, *B* in the periphery of the lens, to be spread uniformly over the line *PS*, the diameter of the circle of aberration; the rays transmitted through every other point in that annulus will be also spread uniformly over lines all equal to *PS*, and intersecting one another in *O*; so that the whole light transmitted through the annulus whose diameter is *AB*, will be spread over a circle whose diameter is *PS*; and the density of the light, in any point of this circle, will be reciprocally as the distance from the centre. Now if the whole circular aperture of the lens be supposed to be divided into an indefinite number of concentrical annuli of evanescent breadth, the light emitted through each will be diffused over a circle whose diameter is in a given ratio to that of the annulus, and therefore is proportional to it; the quantity of light therefore in each circle of aberration, generated by each annulus separately, is as the annulus directly, that is, as its radius; therefore the mean density, or the density in corresponding points in different circles, which is as the quantity of light directly, and inversely as the space it occupies, will be inversely as the radius: and since the density in each circle separately is inversely as the distance from the centre, the density in different circles, at equal distances from the centre, is equal. Let *CD* be the diameter of the

circle of aberration correfpondent to the aperture EF; all the annuli between A and E throw their light on the point C; therefore the denfity of the rays in C, which is as the number of circles intermixed there and the denfity in each conjointly, will be as AE or PC directly, and CO inverfely; that is, the denfity in any point C, is to the denfity in the middle point of the radius, as PC to CO.

22. The whole quantity of light within any interior circle of aberration, is to the quantity of light in the whole circle of aberration, as the difference between the fquares of the radius of the circle of aberration and of the interval between the concentric circles to the fquare of the aforefaid radius.

Fig. 24. For the quantity of light in the annulus of evanefcent breadth whofe radius is OC, is as the annulus and the denfity of light conjointly, or as $OC \times \frac{PC}{OC}$, that is, as PC; the quantity of light therefore in the whole annular fpace whofe breadth is PC, is as the fum of all the PC, that is, as PC^2; therefore the quantity of light in the circle PO is to the quantity of light in the circle CO, as PO^2 to $PO^2—PC^2$.

As the aberration caufed by the unequal refrangibility of the rays is fo great, it might be expected that an image could not be formed fo free from colour as we find it; but from the two laft articles we fee, that the light is not uniformly diffufed over the circle of aberration, but is very rare at the circumference, and very denfe towards the centre; fo that thofe rays only which are near the centre, are ftrong enough to affect the eye.

LECTURE

LECTURE XIII.

1. VISION is performed by the rays of light, which issuing from any point of an object, and falling on the eye, are made by their refraction, in passing through the humours which it contains, to converge to so many points on the back part of the eye, and there paint an image of the object.

The eye is perfectly globular, except that the fore part is a little more convex than the rest. It consists of three mediums or humours; that in the front is a transparent fluid like water, and is therefore called the aqueous humour; the next is called the crystalline humour, which is denser than the aqueous, and in its figure resembles a double convex lens, having its back surface of the greater curvature; the hindermost is the vitreous humour, and its density is less than that of the crystalline. The whole globe, except the fore-part, is surrounded by three coats. The exterior

terior coat, called the sclerotica, is propagated over the whole eye, and is white on the anterior part, except a spherical portion in front, where it becomes transparent, and is called the cornea. Next within, and adherent to this coat, is the choroeides; where it joins the cornea, it is drawn across through the aqueous humour, dividing it into two chambers; this part of the choroeides is called the Uvea or Iris; in its centre is a perforation for the admission of the rays called the pupil; this aperture the eye has a power of contracting or enlarging, for the admission of less or more light, as the circumstances of vision may require. The crystalline is suspended by a muscle called the Processus Ciliares, and sometimes the Ligamentum Ciliare. The third and inmost coat of the eye is the retina; and is a fine expansion of the optic nerve, which taking its origin in the brain, is inserted at the bottom of the eye, and spreads like a network over the inside of the choroeides. The several coats and surfaces of the humours are so situated as to have one straight line, called the axis of the eye, perpendicular to them all.

2. All the humours of the eye contribute orderly to produce a convergence of the rays.

3. The image formed on the retina is inverted.

Though the image is inverted, yet the object appears erect, because inversion is relative; and therefore since the images of all objects preserve their relative position on the fund of the eye, no inversion is perceived.

4. Though there are two images formed, one on

the

the retina of each eye, yet objects do not appear double.

It ſeems that we diſcover the unity of objects, in the ſame manner as we diſcover that they are diſtant, that is, by experience.

5. An object appears ſituated in that place, from which the rays laſt of all diverge in coming to the ſpectators eye.

6. The eye is endued with the power of varying its refractive power.

7. The power of ſeeing diſtinctly at different diſtances does not depend on the chryſtalline.

This is evident from the experiments made on a perſon who had been couched for a cataract, and by the aſſiſtance of the ſame convex lens, applied to that eye, could ſee diſtinctly at different diſtances.

8. The uſe of the chryſtalline ſeems to be to correct the ſpherical errors of the eye, and to increaſe its refractive power.

9. The power of ſeeing diſtinctly at different diſtances does not depend on any change in the general form of the globe of the eye.

See Phil. Tranſ. 1794, Hunter's Lecture.

10. The cornea is made up of laminæ, and is elaſtic.

When ſtretched, it is capable of being elongated $\frac{1}{11}$ part of its diameter, contracting to its former length immediately when the expanding force is removed.

11. The

11. The power of feeing diftinctly at different diftances depends on the change in the figure of the cornea, it being rendered more convex in feeing nearer, than in feeing more remote objects.

This change of figure is rendered manifeft, by obferving the profile of the cornea with a microfcope. This change is effected by the tendons of the four ftraight mufcles of the eye, which are continued on to the edge of the cornea, and terminate or are inferted in its external lamina, fo that its action extends fo far.

12. The diftance of objects is judged of by various circumftances, which take place in various cafes.

The change of conformation in the cornea is one of the means by which we judge of fmall diftances, for this, being an animal action, is capable, in confequence of habit, of fuggefting to the mind the different diftances of objects. In optical inftruments, where the images of objects are viewed with one eye, this information is probably the caufe which fuggefts the diftance of thefe images; and that the eye fees them in that place whence the rays proceed, which enter the eye from the feveral parts of the image. But this method of eftimating diftance will not ferve us much beyond twenty or thirty inches. 2. The inclination of the Optic axes of the eyes is another more certain method of eftimating diftance. This method extends to about five or fix feet; beyond that extent the differences of the optic angles are fo fmall as to become infenfible. That this caufe really operates, appears from this, that we judge of

of distance more accurately with both eyes, than with one only. 3. The length of the ground plane, or the number of intervening objects perceived in it. Hence an ascending plane appears longer than a level plane of the same extent. 4. The angle under which an object appears, whose magnitude is supposed to be known; or if in motion, though not known, the variation of that angle. 5. All other things being the same, different colours and degrees of brightness of objects suggest a difference of distance.

13. The aberration arising from the different refrangibility of the rays of light on the retina is so small as to be imperceptible.

Dr. Maskelyne, in the Philosophical Transactions for 1789, has computed the diameter of the circle of aberration upon the retina, and found it to be ,002667 of an inch, a quantity too small to be perceived.

14. The natural defects of sight are either that the rays converge behind the retina, or before they arrive at it.

Those who labour under the first species of defective sight are called Presbytæ; those who labour under the latter are called Myopes.

15. Myopes correct the defect in their sight by a concave lens; Presbytæ by a convex lens.

Fig. 25, 26. The radius of curvature of the convex or concave lenses for these purposes should be equal to the product of the distances of distinct vision from the naked and armed eye, divided by their difference. Thus let LC be the

least distance with which a long sighted person, or the greatest at which a near-sighted person can see distinctly, and it be required to find FC the radius or focal length of a double convex or double concave lens, which shall enable him to see distinctly at the distance AC: let F be the principal focus of the lens; then $AF : AC :: AC : AL$, therefore $AF \pm AC$ or $CF : AC :: AC \pm AL$ or $LC : AL$, that $CF = \frac{AC \times LC}{AL}$.

16. If a person can see an object distinctly at two distances whereof one is double the other, an equal alteration made in the refractive power of the eye, will enable him to see the object distinctly at an infinite distance.

Hence a short-sighted person can see distinctly at *all distances* with a single concave of a proper figure.

17. The least angle which the parts of a compound object distinctly visible subtend at the eye, is about $4' \, 5''$: and the least angle subtended by a simple black object on a white ground is about $2'$, at a medium.

To the generality of eyes, the nearest distance of distinct vision is about 7 or 8 inches; hence if we take 7 inches for that distance, and $2'$ for the least visible angle, a globular object, of less than about the $\frac{1}{150}$ part of an inch, cannot be seen.

LECTURE

LECTURE XIV.

1. IF a convex lens be exposed to different radiants, not very remote from each other, their images will be formed with sufficient accuracy, but inverted, on a screen placed in their focus.

On this principle is constructed the camera obscura. If the focal length of the lens be too great, the image will not be sufficiently bright; if it be too short, the difference between the focal distances of the colours will be so great as to render the picture confused.

2. If a minute object be placed in the principal focus of a lens of very small focal length, when seen through the lens, it will appear magnified and distinct.

This is the single microscope. The angle under which the object appears, will be to that which it subtends when

feen by the naked eye, as the diftance at which it is viewed by the naked eye diftinctly, to the principal focal length of the lens.

3. A minute object, placed at the diftance of half a radius from the furface of a very fmall glafs fpherule will appear magnified and diftinct.

The magnifying power of fuch a fpherule is equal to that of a lens, whofe focal length is equal to a femidiameter and an half of the fpherule.

4. If an object be placed a little beyond the principal focus of a convex lens, there will be formed a large inverted image in the conjugate focus, and if another convex lens be placed at its focal diftance from this image, it will appear very much magnified and diftinct.

This is called a compound microfcope; the lens turned to the object is called the object-glafs, and that next the eye is called the eye-glafs. In this microfcope, the linear magnifying power is equal to the leaft diftance of diftinct vifion multiplied by the diftance of the image from the object-glafs, divided by the product of the diftance of the object from the object-glafs into the focal length of the eye-glafs. The object appears inverted, becaufe the eye looks at an inverted image. The brightnefs of the object is as the area of the object glafs; and the field of view as the area of the eye-glafs, all other circumftances being the fame.

5. If an object be placed between the principal focus and centre of a concave fpeculum, there will be

be formed on the other ſide of the centre an enlarged image, and if a convex lens be placed at its focal length from this image, it will appear magnified and diſtinct.

This is called a reflecting microſcope. The angle under which the object appears to the naked eye, is to the angle under which it appears in this microſcope, as the rectangle under the diſtance of the object from the ſpeculum, and the focal length of the lens, to the rectangle under the diſtance of the image from the ſpeculum, and the leaſt diſtance of diſtinct viſion.

6. If a minute, tranſparent object, illuminated by the rays of the ſun or a lamp, be placed before a ſmall lens, at a little greater diſtance from it than the principal focus, the image will be deſcribed on a ſcreen, placed in the conjugate focus, diſtinct and magnified.

On this principle is conſtructed the Solar Microſcope, and Magic Lantern.

LECTURE

LECTURE XV.

1. IF a broad convex lens be turned towards a very remote object, its image will be formed very nearly in the principal focus; and if between the image and the object glass there be interposed a concave eye-glass, whose distance from the image may be equal to its own focal length, the object, when seen through this combination of lenses, will appear distinct and magnified.

This is called the Gallilean telescope. Its length is equal to the difference between the focal lengths of the lenses. Its magnifying power is equal to the focal length of the object-glass, divided by that of the eye-glass. It shews objects erect, because the eye-glass intercepts the rays before the image is formed by the object-glass; and the visible area or field of view is as the magnitude of the pupil of the eye, and

and will alſo be greater, the nearer the eye is to the glaſs; all other circumſtances being the ſame.

2. If a broad convex lens be turned towards a very remote object, its image will be formed in the principal focus; and if a ſmall convex lens be placed at its own focal diſtance from this image, the object ſeen through both lenſes, will appear diſtinct and magnified.

This is called the Aſtronomical Teleſcope. Its length is equal to the ſum of the focal lengths of the lenſes. Its magnifying power is equal to the focal length of the object-glaſs, divided by that of the eye-glaſs. It ſhews objects inverted, which however is not attended with any inconvenience in aſtronomical obſervations; and the field of view is directly as the breadth of the eye-glaſs, and inverſely as the interval between the lenſes.

The image may be rendered erect by the addition of two eye-glaſſes more; one of which is placed at twice its focal length from the firſt eye-glaſs, and the third or principal eye-glaſs at its own focal length from the ſecond image. A teleſcope thus conſtructed with four glaſſes is called the Terreſtrial Teleſcope. The eye glaſſes are generally of the ſame focal length, in which caſe it magnifies as the aſtronomical teleſcope.

3. The perfection of dioptric teleſcopes is impeded by the different refrangibility of light, and the ſpherical figure of the lenſes.

4. The

4. The imperfection of the object-glafs arifes principally from the different refrangibility of light; and of the eye-glafs, from its fpherical figure.

5. If the rays iffuing from a very remote object, fall parallel to the axis on a concave fpeculum, and being intercepted by a plane fpeculum, which forms an angle of 45° with the axis, before they come to their focus, be reflected to a convex lens, placed at its focal length from the image, the object will appear diftinct and magnified.

This is the Newtonian telefcope. Its length is equal to the focal length of the fpeculum. Its magnifying power is equal to the focal length of the fpeculum divided by the focal length of the eye-glafs. It fhews objects inverted, and the field of view is directly as the linear aperture of the eye-glafs, and inverfely as the focal length of the fpeculum.

James Gregory is generally fuppofed to have been the firft who conceived the idea of a reflecting telefcope, of which he has given a delineation in his Optica Promota, publifhed in the year 1695. But Newton certainly was the firft perfon who demonftrated both its importance and practicability. And even as to the honour of having firft conceived the idea of this excellent inftrument, Gregory muft refign it to the Jefuit Efkinard, who appears, fo early as the year 1615, to have diftinctly defcribed it in his Century of Optical Problems. It is alfo to be remarked, that Gregory propofed the reflecting telefcope merely to

remedy

remedy the ſpherical errors of lenſes; but the object-glaſſes of teleſcopes are too ſmall a portion of a ſphere to make the defects, ariſing from their figure, ſenſible.

6. If rays from a very remote object fall perpendicularly on a large concave ſpeculum, perforated in the centre, and after forming an image, they be reflected back in a contrary direction from a ſmall concave, ſo as to form a ſecond image, the firſt image lying between the centre and principal focus of the leſſer ſpeculum, the object ſeen through a convex lens, placed at its focal diſtance from the ſecond image, will appear diſtinct and magnified.

This is the Gregorian teleſcope. It ſhews objects erect, becauſe the number of real images is even. Its magnifying power is as the product of the focal length of the great ſpeculum and diſtance of the ſecond image from the principal focus of the ſmaller ſpeculum, divided by the product of the focal lengths of the ſmaller ſpeculum and eye-glaſs. The field of view is equal to the angle which the eye-glaſs placed in the focus of the great ſpeculum ſubtends at the centre, diminiſhed in the ratio of the diſtance of the ſecond image from the principal focus of the ſmaller ſpeculum to its focal length.

The Caſſegrain conſtruction is the ſame with the Gregorian, except in the form and poſition of the ſmall ſpeculum, which is convex, inſtead of being concave; and is placed before the principal focus of the great concave,

not behind it, as in the Gregorian. The principal objection to this construction consists in the difficulty of giving the true form to the small speculum. For in the Gregorian, the great speculum ought to be parabolic and the little speculum elliptic: in the Cassegrain the great speculum ought also to be parabolic, but the little speculum hyperbolic. Now in grinding all sorts of concave speculums and lenses, it is found by experience, that they vary from the spherical to the parabolic form, or even go beyond it; and that all convex surfaces vary from the spherical form towards that of an oblate spheroid, next to one of its poles. Hence the variations from the spherical form, which arise from the manner of working them, in both speculums of the Gregorian telescope, lie the right way to correct the errors: but in the little speculum of the Cassegrain form, *they lie* the wrong way, and tend to increase instead of diminishing or correcting them.

7. In telescopes of the same length, the magnifying power of a reflector is much greater than that of a refractor.

LECTURE

LECTURE XVI.

1. IF the ſine of incidence be to the ſine of refraction of the rays of mean refrangibility, paſſing out of air into any medium, as $m : 1$; of the violet rays as $m + \dot{m} : 1$; and of the red, as $m - \dot{m} : 1$; the quantity $\dot{m}$, being conſtant in the ſame medium, is aſſumed as a meaſure of the diſſipating power.

Thus the ſine of incidence is to the ſine of refraction in the rays of mean refrangibility, paſſing out of air into common glaſs, as $\frac{77{,}5}{50} : 1$; in the red rays as $\frac{77}{50} : 1$; and in the violet as $\frac{78}{50} : 1$; wherefore putting $\frac{77{,}5}{50} = m$, we have $\frac{78}{50} = m + \dot{m}$; and $\dot{m} = \frac{1}{100}$.

If the ſine of incidence of red rays, paſſing out of air into flint glaſs, be to the ſine of refraction as 1,565 to 1;

of the violet as 1,595 to 1; then the ratio of refraction in the rays of mean refrangibility is as 1,583 to 1; if $n = 1{,}58$, $\dot{n} = \frac{3}{200}$; and the ratio of the dispersive powers of flint and common glass will be as $\frac{3}{200} : \frac{1}{100}$, or as $3 : 2$.

2. If a ray of solar light be refracted by one surface of a refracting medium, let $t =$ the tangent of refraction of the rays of mean refrangibility, $m : 1$ the ratio of refraction of the same rays, $\dot{m}$ the measure of the dissipating power; then will the angle of dispersion be subtended by an arc $= \frac{2\dot{m}t}{m}$, radius being $= 1$.

Fig. 27. For let FQ be the incident and QR the refracted ray; $m : 1 :: \text{cos. } p : \text{cos. } q$, therefore $m \text{ cos. } q = \text{cos. } p$, where q and m are variable, and p constant; therefore $\dot{m} \text{ cos. } q - m\dot{q} \text{ sin. } q = 0$; and $m\dot{q} = \frac{\dot{m} \text{ cos. } q}{\text{sin. } q}$, and $\dot{q} = \frac{\dot{m} \text{ cos. } q}{m. \text{ sin. } q} = \frac{\dot{m}t}{m}$; where $\dot{q}$ denotes the variation of the angle q; and as this is equal on both sides of the mean ray, on one side by the red, and on the other by the violet, the whole dispersion will be equal to $\frac{2\dot{m}t}{m}$.

Thus if a solar ray impinge on a surface of common glass at an angle of incidence $= 20°$; then will the angle of refraction of the rays of mean refrangibility $= 12° \, 44' \, 52''$, $m = \frac{77{,}5}{50} = 1{,}55$ and $\dot{m} = \frac{1}{100}$, therefore the angle of

dispersion

dispersion $= \frac{2 \text{ tang. } 12^\circ\, 44'\, 52''}{100 \times 1,55} = 10'\, 3''$, radius being 1.

See Euler's Dioptrics, and Atwood's Analysis.

3. The powers of dissipation and refraction of different refracting substances may be thus determined: suppose a solar ray to pass through a prism, the refracting and dissipating powers of which are required, impinging perpendicularly on the first surface; let the angle of dissipation, and the angle of refraction of the middle rays be accurately measured, and let d = the measure of the angle of dissipation, t = the tangent of refraction, radius being = 1; the angle of incidence on the second surface will be = the refracting angle of the prism, and the ratio of refraction is determined by measurement; if this be as $n : 1$, then the measure of the dissipating power $\dot{n} = \frac{dn}{2t}$.

For $d = \frac{2\dot{n}t}{n}$, and therefore $\dot{n} = \frac{dn}{2t}$.

4. If a ray of solar light be refracted through two surfaces, inclined to each other, as are the sides of a prism, whose refracting angle is a; and if the complement of the angle of refraction at the first surface be q, and the complement of the angle of refraction at the second surface = r, then will the angle of dissipation, at which the violet and

and red rays are inclined to each other, after refraction at the second surface, be $= \frac{2\dot{m} \text{ sin. } a}{\text{sin. } q \times \text{sin. } r}$.

Fig. 28. For if p be the complement of the angle of incidence on the first surface, then $m : 1 :: \text{cos. } p : \text{cos. } q$, and therefore $m \text{ cos. } q. = \text{cos. } p.$ and $1 : m :: \text{cos. } \overline{q - a} : \text{cos. } r$, and therefore $m \text{ cos. } \overline{q - a} = \text{cos. } r$; in which equations, q, r, and m are variable, and p and a constant; then taking the fluxions of these equations, $\dot{m} \text{ cos. } q - m\dot{q} \text{ sin } q = 0$, and $\dot{m} \text{ cos. } \overline{q - a} - m\dot{q} \text{ sin } \overline{q - a} = -\dot{r} \text{ sin. } r$. From the first equation, $m\dot{q} = \frac{\dot{m} \text{ cos. } q}{\text{sin. } q}$; and from the second $\dot{m}$ $\text{cos. } \overline{q - a} - \frac{\dot{m} \text{ cos. } q \times \text{sin. } \overline{q - a}}{\text{sin. } q} = -\dot{r} \text{ sin. } r$. But sin. $q \times \text{cos. } \overline{q - a} - \text{cos. } q \times \text{sin. } \overline{q - a} = \text{sin. } \overline{q - \overline{q - a}} = \text{sin. } a$; therefore $\frac{\dot{m} \text{ sin. } a}{\text{sin. } q} = -\dot{r} \text{ sin } r$, and $\dot{r} = \frac{-\dot{m} \text{ sin. } a}{\text{sin. } q \times \text{sin. } r}$. where $\dot{r}$ denotes the variation of the angle r. And the whole dispersion, when $m - \dot{m}$ is substituted for the red rays, and $m + \dot{m}$ for the violet, will be $= \frac{2\dot{m} \text{ sin. } a}{\text{sin. } q \times \text{sin. } r}$.

Thus, if a solar ray impinge on a flint prism, whose refracting angle is 19°, at an angle of incidence = 16° 31′ ¼, it will emerge at an angle of refraction = 50° 48′ 18″; here $\dot{m} = \frac{3}{200}$, and therefore the angle of dissipation after emergence $= \frac{3 \times \text{sin. } 19°}{100 \times \text{cos. } 10° 22' 26'' \times \text{cos. } 50° 48' 18''} = 54'$, radius being $= 1$.

In

In like manner, if a ray of solar light be refracted through a prism of common glass, whose refracting angle is 30°, when the rays of mean refrangibility pass parallel to the base, the angle of dissipation will be 39′, for here $m = \frac{1}{100}$, the angle of refraction at the first surface = 15° or half the vertical angle of the prism, and the angle of refraction at emergence = 23° 39′ 5″; therefore the angle of dissipation $= \frac{2 \times \text{sin. } 30°}{100 \times \text{cos. } 15° \times \text{cos. } 23° \; 39' . \; 5''} = 38'$ 51″.

If the rays fall perpendicularly on the first surface, the cos. of refraction at the first surface = 1, and the angle of refraction at emergence = 50° 48′ 18″, therefore the angle of dissipation $= \frac{2. \text{ sin. } 30°}{100 \times \text{cos. } 50° \; 48' \; 18''} = 54' \; 24''$.

5. Two prisms made of different kinds of glass may have their refracting angles so adjusted, that when the refracting angle of one is applied to the base of the other, a ray of light passing through them shall have its incident and emergent parts parallel, and the emergent part shall be coloured.

This arises from the difference between the dispersive and refractive powers in different kinds of glass. Thus if the vertex of a flint glass prism, the refracting angle of which is 23° 40′, be applied to the base of a common glass prism, the refracting angle of which = 25°, a ray of solar light will pass directly through the prisms, when their surfaces are contiguous, but the emergent ray will be coloured. The emergent ray is parallel to the incident ray, because

cause in the given circumstances, the mean refractive powers are equal and contrary; but it is coloured, because in the same circumstances, the dispersive powers are unequal.

6. Two prisms may be applied as before, and the emergent ray shall be free from colour, but not parallel to the incident ray.

For as the mean refractions may be equal and contrary, and the dispersions unequal, so the dispersions may be equal and contrary, and the mean refractions unequal.

Thus if the vertex of a common glass prism, whose refracting angle is 30°, be applied contiguous to the base of a prism of flint glass, the refracting angle of which is = 19°. a solar ray being refracted through them will deviate from the course of the incident ray, but will not be separated into the coloured rays.

7. Three prisms of different kinds of glass may have their refracting angles so adjusted, that when the refracting angle of the intermediate prism is applied contiguous to the bases of the two extreme ones, a solar ray being refracted through them shall emerge colourless, and yet deviate from the course of the incident ray.

Thus if the vertex of a flint glass prism, whose refracting angle is 23° 40′ be applied contiguous to the bases of two prisms of common glass whose refracting angles are respectively 25° and 10°, a ray of solar light passing through the three prisms, and emerging at an angle of 16° 57′ will deviate about 5° 37′ from the course of the incident ray, and

and will be colourlefs. For the two common glafs prifms refracting the ray in the fame direction, would caufe it to deviate from the courfe of the incident ray, about 5° 37′ more than the deviation in the contrary direction arifing from refraction through the flint prifm; but the latter by its greater diffipating power, exactly counteracts the feparation of the rays occafioned by refraction through the other two prifms.

8. The refracting and diffipating powers of two lenfes, one convex of common glafs, the other plano-concave or concavo-convex of white flint, being given, the radii of the furfaces may be fo adjufted to each other, that the extreme, principal, and intermediate images fhall coincide.

The two lenfes muft act on the rays of light in the fame manner as two achromatic prifms, and therefore their refractions muft be made in contrary directions, that is, the one muft be convex, and the other concave; and as the rays are to converge to a real focus, the excefs of refraction muft be in the convex lens. Farther, as the convex lens is to refract moft, it muft be made of crown glafs, whofe refractive power, in equal difperfions, is greater than that of flint. For in equal refractions, the difperfive power of flint glafs is to that of crown, as 3 to 2; and the fine of incidence is to that of refraction, of the mean rays, in flint glafs, as 1,583 to 1; and in crown glafs, as 1,53 to 1; the difperfive power therefore of flint glafs exceeds that of crown glafs, in a greater ratio than its mean refraction exceeds the mean refraction of crown glafs; and therefore when its difperfion is equal to that of crown glafs, its re-

fraction will be lefs. A compound lens thus conftructed, is called the double object glafs.

9. The aberration arifing from the fpherical figure of the lens is not entirely corrected in the double object glafs.

10. An object glafs may be compounded of three lenfes, whereof two are double convex, made of common glafs, enclofing a double concave of flint glafs, fo that the extreme and principal images of objects formed by it fhall coincide.

In the double object glafs the refraction of the convex lens being greater than that of the concave, the aberration arifing from its fpherial figure is alfo greater than that of the concave; but in the triple object glafs, the refraction of the common glafs being equally divided between two lenfes, the whole aberration of the convex lens is fo far diminifhed as to become equal to that of the fingle concave lens.

11. The refracting and diffipating powers of the three lenfes which conftitute the triple object glafs being given, the radii of the furfaces may be fo adjufted to each other, that the extreme, principal, and intermediate images of objects formed by it, fhall be diftinct and colourlefs.

The triple object glafs corrects the difperfion of the rays on the fame principle with the three achromatic prifms; and at the fame time the aberration arifing from the fpherical figure is wholly removed. Object glaffes thus constructed

Fig. 1.

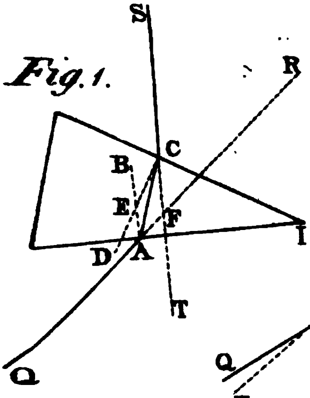
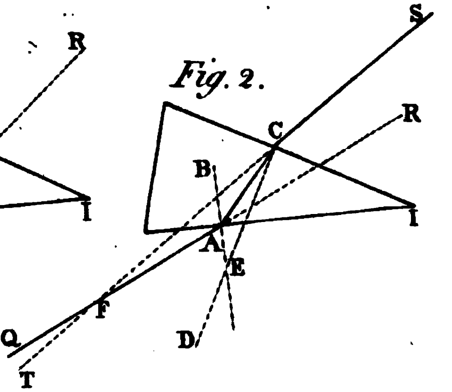


Fig. 2.



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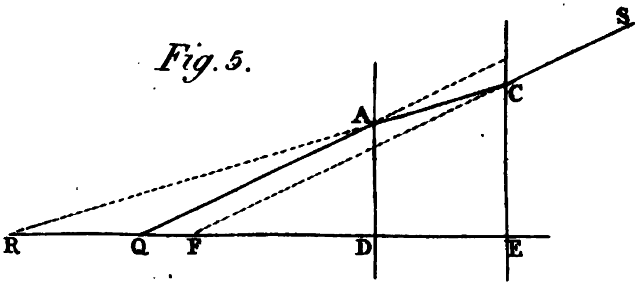

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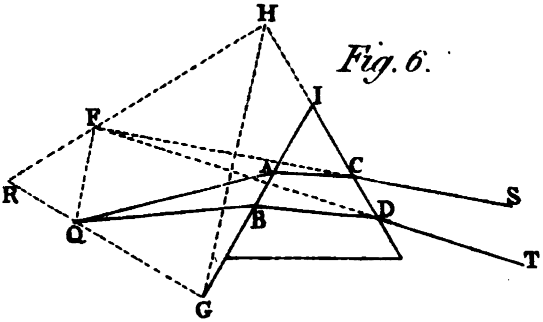

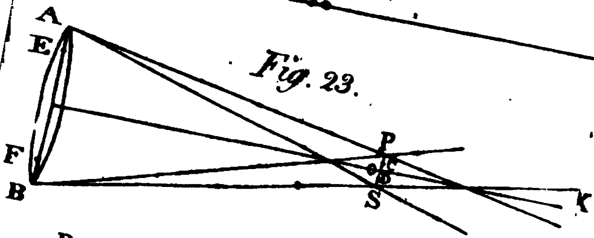

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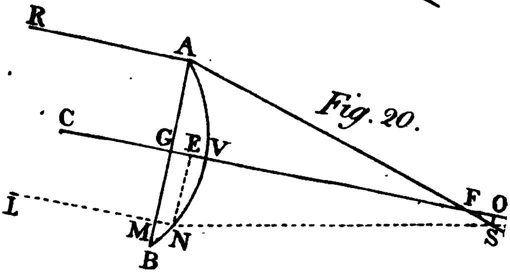


Fig. 20.

Fig. 17.

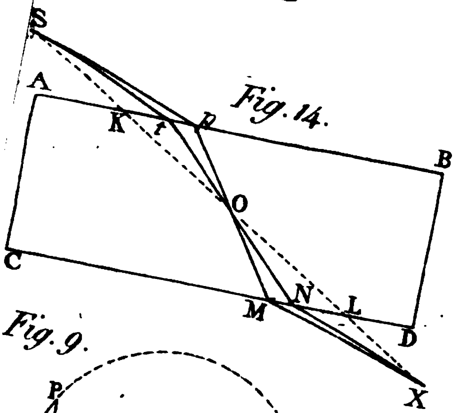


Fig. 14.

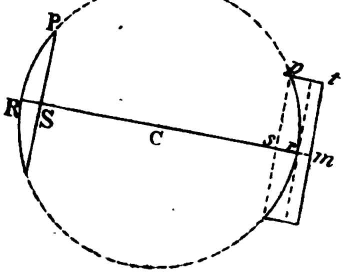


Fig. 9.

9.

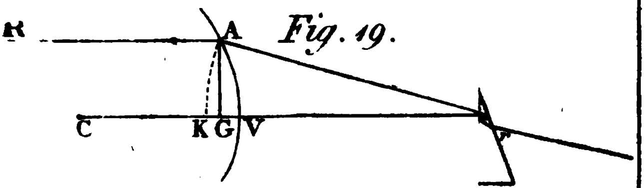


Fig. 19.

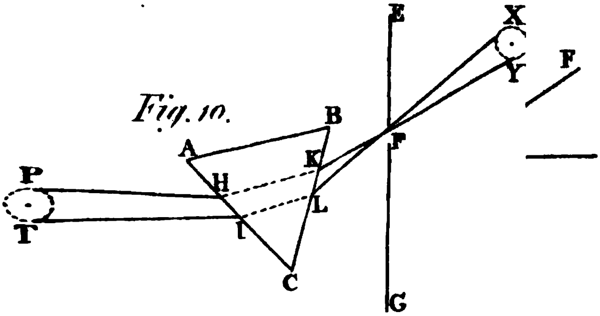


Fig. 10.

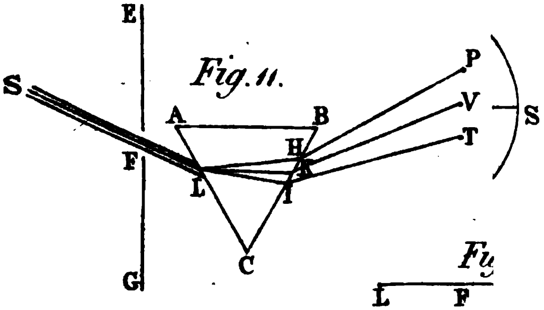


Fig. 11.

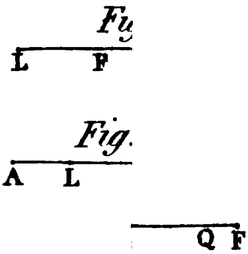


Fig.

ed are called perfect lenſes. But the double object-glaſs ſeems preferable, as ſo much more light is loſt in the triple object glaſs at the two additional ſurfaces.

12. Teleſcopes conſtructed with achromatic object glaſſes, are better than reflecting teleſcopes.

More light is loſt by reflection than by refraction through a triple object glaſs, and any error in the figure produces ſix times a greater effect in reflection than refraction: hence the image formed by reflection is not ſo diſtinct and ſharp as by refraction.

13. The indiſtinctneſs of the image ſeen through a teleſcope, occaſioned by the diſperſion at the object glaſs, is the ſame in every part of the field; but that which is occaſioned by the eye-glaſs is different according to the part of the field in which it is ſeen.

The pencils which iſſue from each point of the object are diffuſed over the whole object glaſs, and therefore the diſperſion of the image of every point of the object is the ſame, in whatever part of the field it may be. But the pencils which iſſue from the image formed in the focus of the object glaſs are not diffuſed over the entire eye-glaſs, each pencil falling nearer to the verge of the eye-glaſs, the farther it is from the centre of the image. Hence alſo it is, that varying the aperture of the eye-glaſs varies the field, whereas varying the aperture of the object glaſs does not.

14. If the plane ſide of a plano-convex eye-glaſs be placed near the image formed by the object glaſs, the rays will emerge free from any colours generated by that eye-glaſs.

15. The errors ariſing from the ſpherical figure of the eye-glaſs may be corrected. 1. By increaſing the number of eye-glaſſes. 2. By giving the glaſſes, eſpecially thoſe which are concerned in forming the laſt image, large focal diſtances and ſmall apertures.

ELECTRICITY.

ELECTRICITY.

Of Electricity in General.

1. ELECTRICITY is that power by which bodies, under certain circumſtances, attract, and after contact repell light ſubſtances placed near them.

Thus, take a glaſs tube an inch and an half in diameter, and about three feet long; rub this tube, from one end to the other, with a piece of dry ſilk, and it will exhibit Electric appearances.

2. The

2. The caufe of electrical phænomena is a body.

For it generates found in its paffage from one body to another, produces light, heats and even melts metals, generates a phofphoric fmell; excites pain &c; it is therefore poffeffed of properties, and is confequently a body.

3. The caufe of electrical phænomena is a fluid.

For when a quantity of electric matter is communicated to a body, every part of which is equally capable of receiving it, it diffufes itfelf equally throughout that body.

Thus if a body be placed on an electric ftand and communicate with the prime conductor, every part of it will equally exhibit electrical phænomena.

4. The electric fluid is attracted by all bodies, but by different bodies in different degrees.

This follows from its being a body.

5. The electric fluid appears to be heterogeneous.

Firft, in general, in combuftion the prefence of oxygen is neceffary; but in the electric difcharge, light and heat are both extricated without the aid of oxygen; for when the difcharge is made in oxygen air, the air fuffers no change either in bulk or quality, but folely in an increafe of temperature: alfo, the electric difcharge can be made in fixed air, or in perfect azotic gas; confequently the light and heat, manifefted in the electric difcharge, are extricated from the electric fluid; therefore fince the capacity of the electric

electric fluid for retaining light and heat is in this case diminished, this effect must proceed from the union of different gases, whose capacity for retaining light and heat, when they are combined together, is less than when they are separate. This is agreeable to an experiment, mentioned by Mr. Kirwan, which was made in Holland: filings of copper, mixed with sulphur, were put into a small glass vial, out of which the common air was, in some cases, exhausted, and in others the vial was filled up either with fixed or mephitic air; it was then heated over burning coals; upon which the mixture swelled, some sulphur sublimed, and the metal became red hot, and, in some cases, even with inflammation; on which Mr. Kirwan makes the following observations, "In these circumstances, I think," says he, "the ignition proceeded from the excess of the specific heat of the sulphur and copper before their union, "over that retained by them after their combination." See Miner. vol. 2. p. 509. When the electric discharge is made in atmospheric air, there follows a considerable diminution of bulk; but this does not arise from the absorption of any element of atmospheric air by the electric fluid; but by the union which is formed, by means of the increased temperature, between the bases of oxygen and azotic gas; whence true nitre is formed, as appears from the experiments of Mr. Cavendish. After the electric discharge has been often made in atmospheric air, it is true that the residuum is highly noxious; but this does not arise from any carbon extricated in the electric combustion, nor from any absorption of oxygen by the electric fluid; but merely from the generation

generation of the nitrous acid, in which oxygen and azot are mixed in the ratio of 5 to 3; ſo that the oxygen being diminiſhed in ſo much an higher proportion than the azot, the azot, towards the end of the experiment, becomes very predominant. 2. The electric ſpark taken in any kind of oil, produces inflammable air. Now oil conſiſts of hydrogen, carbon, and oxygen, without the intervention of caloric to bring the hydrogen and oxygen into the ſtate of gas. See Lav. p. 166; it is therefore probable, that the electric fluid parts with caloric in the diſcharge, which uniting with the hydrogen forms inflammable air. 3. In excitation, a phoſphoric ſmell is produced; which is probably cauſed by ſome of the volatile particles which eſcape undecompoſed, in the ſeparation of the two gaſes, of which the electric fluid conſiſts. 4. Inflammable air is fired by the electric diſcharge. 5. Metals are made red hot, and even melted. 6. Metals are oxygenated by the diſcharge; but they diſoxygenate the air in conſequence of their temperature being increaſed; the increaſe of temperature therefore is previous to the extrication of caloric from it, and therefore the metals muſt be heated by the caloric extricated from the electric fluid.

6. The electric fluid is received from the earth.

For if the communication of the electric machine with the earth be cut off by placing it on non-conductors, it will quickly ceaſe to exhibit electrical appearances.

7. The electric fluid is compreſſible.

Let a metallic cup be placed on an electric ſtand; lay in the cup a pretty long metallic chain, having a ſilk thread

tied

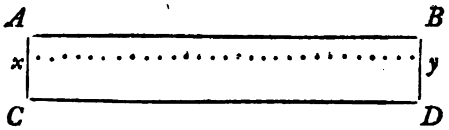
tied to one end; from the handle of the cup suspend a cork ball electrometer; then electrify the cup, and the balls of the electrometer will diverge. If in this situation, one end of the chain be raised up above the cup, by the silk thread, while the other end remains in it, the balls of the electrometer will converge a little, and more in proportion as the chain is raised higher; which proves that the electric matter is more dense, when these bodies are in a compact, than when in a more extended form.

On this principle is constructed the electrical condenser.

The contrary of this article is maintained by M. Marat, see his Recherches sur l'Electricitè, p. 35—40; but Van Swinden does not think that his experiments invalidate the position here laid down.

8. The energy of electric action varies in some inverse ratio of the rarity of the electric matter.

9. The energy of electrified bodies is proportional to their surfaces, not solid contents.



For let *ABCD* be the electrified body, and *Ax* the space through which the attraction of the body on the electric matter acts; any particle of the electric matter at *x*, or below it, will have equal attractive forces above and below it, which therefore destroy each other's effects. The force of the body therefore on the electric fluid within

the body, is proportional to the plate $AxyB$ of a given altitude, that is, proportional to the ſurface of the body.

The force of the body on the external electric fluid is alſo manifeſtly proportional to the ſame plate.

10. Electrified bodies manifeſt their energy through thin electric plates.

11. The electric fluid and lightning are the ſame ſubſtance.

12. The Aurora Borealis is not an electrical phænomenon.

For the electrical kite is not affected by it. See Cavallo, and alſo Van Swinden's Mem. vol. 3.

13. There is an analogy between the electric matter and heat.

For, 1. They are both excited by friction. 2. The beſt conductors of electricity are alſo in general the beſt conductors of heat. 3. The electric matter is luminous. 4. It produces inflammation. 5. Fire promotes vegetation; ſo does electricity. See Adams's Elect. p. 495.

14. The electric matter and heat are different.

For, 1. Some good conductors of electricity, as water, are bad conductors of heat. 2. The electric matter has always the ſame odour, whatever may be the nature of the electrified body; not ſo with common fire. 3. The electric matter penetrates the denſeſt bodies almoſt inſtantaneouſly; fire ſlowly. 4. Metallic ſubſtances equally fuſible by electricity, are not ſo by fire; and iron which is leſs fuſible by fire than gold, is much more ſo by the electrical exploſion.

Of

Of Conductors.

15. The electrical fluid paſſes along or through ſome bodies with great facility, through others ſlowly, and with great difficulty.

This is agreeable to the analogy of heat or caloric. The former claſs of bodies are called conductors, the latter non-conductors. Thus the electric fluid of the prime conductor may be communicated to another body by means of a rod of any metal, but not by a rod of glaſs. The latter claſs of bodies are alſo called electrics, becauſe they are capable of exhibiting electrical phænomena; the former, on the contrary, are called non-electrics. An electric when in a ſtate of exhibiting electrical phænomena is ſaid to be excited. When it communicates this power to a conductor, the latter is ſaid to be electrified.

16. Conducting ſubſtances are ſuch in different degrees.

Thus the electric fluid will paſs more readily over a metal rod, than one of wood.

Though all bodies are divided into conductors and non-conductors, nevertheleſs, in accuracy, perhaps there is no ſuch thing in exiſtence as a perfect conductor or non-conductor; there is an infinity of gradations; and it is, properly ſpeaking, by the quantities that prevail, and which prevail in a conſiderable degree, that bodies are claſſed in this caſe, as well as all others in Natural Philoſophy.

17. Glafs is not abfolutely a non conductor.

This appears from the fpontaneous difcharge, and other experiments.

18. Glafs is not abfolutely impervious to the electric fluid.

Becaufe it is not abfolutely a non-conductor: a violent charge will indeed break it; but a weak charge, if concentrated in a point, may be tranfmitted through it, without breaking it. The like happens with refpect to conductors alfo, a charge of a certain magnitude may be tranfmitted through them without damage; but it may be fo great as to rend the conductor in pieces.

19. Air rarefied to a certain degree is a conductor.

Mr. Morgan, who has made fome excellent experiments on the non-conducting power of a perfect vacuum, obferves, that there feems to be a limit in the rarefaction of air, which fets bounds to its conducting power; or, that the particles of air may be fo far feparated from each other, as no longer to be able to tranfmit the electric fluid; that if they are brought within a certain diftance of each other, their conducting power begins, and continually increafes, till their approach alfo arrives at its limit, when the particles on the other hand, become fo near as to refift the paffage of the fluid entirely, unlefs by violence, which is the cafe in common and condenfed air. Adams p. 318.

20. Good conductors, as metals, are rendered worfe, and non-conductors, as glafs and refin, are rendered,

rendered, in ſome degree, conductors, by pulverifation.

Becauſe the conſtituent particles of the body are not in contact after pulveriſation; and therefore a plate of air is interpoſed between the adjacent particles; but air is a worſe conductor than metals, and a better conductor than glaſs or reſin.

21. The conducting quality of all bodies ſeems to depend on their degree of diſoxygenation.

Thus metals and charcoal, while diſoxygenated, conduct; when oxygenated they do not. It may perhaps ſeem contrary to this hypotheſis of Dr. Prieſtly, that acids are good conductors: but we are to obſerve that their ſubſtances are not ſaturated with oxygen; on the contrary they have ſtill a very great affinity for that element.

Perhaps the conducting or non-conducting quality of bodies depends on the leſs or greater attractive force with which the electric matter is held by the conſtituent particles of the body; and that heat diminiſhes this attractive force; and thus renders thoſe bodies conductors, which before were not ſuch. See Morgan's Lectures on Electricity, Vol. 2. p. 176.

22. Non-conducting ſubſtances are rendered conductors by being ſufficiently heated; and conductors, by being ſufficiently cooled, become non-conductors.

This is conformable to the laſt article, for an inflammable

ble body by being heated has its attraction for oxygen increaſed, Lavoiſier, p. 256. Alſo metals become worſe conductors by having the electric diſcharge frequently made through them, becauſe they become more oxygenated. Adams p. 276. The degree of temperatûre at which different bodies become conductors is probably different, for the ſame reaſon that they are combuſtible, or become capable of diſoxygenating the air at different temperatures. Thus charcoal which conducts electricity at the common temperature, is alſo combuſtible at the ſame temperature; whereas the diamond, which does not conduct, requires a very great heat for combuſtion, and moſt probably juſt previous to combuſtion becomes a conductor.

23. The electric matter always requires a conductor to enable it to paſs from one body to another.

For it cannot be tranſmitted through a Torricellian vacuum. See Nicholſon's Nat. Phil. vol. 2. p. 318.

24. The electric matter is communicated rather along the ſurface of a conductor, than through its ſubſtance.

For the conducting quality of bodies ſeems to depend on the degree of force with which they attract the electric matter, (ſee Art. 21.) now the particles of the body which by their attraction can generate motion in the electric fluid, are thoſe ſuperficial particles only whoſe depth within the body is equal to the indefinitely little ſpace through which the attractive force extends.

And

And though the conducting power of bodies is not diminifhed, when they are coated with a non-conducting fubftance; yet it is probable, that this coating is not abfolutely in contact with the conductor. Neverthelefs, it is not to be doubted, but that where a great quantity of electricity is made to pafs along a very fmall wire, it will enter the fubftance of the metal. See Cavallo, vol. 1. page 330.

25. If an electrified body be furrounded by perfect non-conductors, its electricity will be permanent.

A body thus circumftanced is faid to be infulated. As the air is not a perfect electric, electrified or excited bodies, though fupported on non-conductors, will continually impart fome of their electricity to the air, or to the conducting particles which float in it; till at laft they entirely lofe their power.

Of Excitation.

26. Bodies by their contact with others, have their natural capacity for retaining certain elements of the electric fluid in fome cafes increafed; and for retaining the others, at the fame time, dimidifhed, and therefore decompofe the fluid.

This is agreeable to the analogy of many chemical facts. Mr. Adams tells us, p. 183 of his Electricity, that mar-

ble

ble or wood varniſhed and heated, by merely laying a metal plate on them, or marble well poliſhed and heated, though not varniſhed, on which a metal plate is cloſely preſſed, will exhibit ſigns of electricity.

27. The electric particles of the ſame kind ſeem to repel each other, and to attract thoſe which are of a different kind with very great force.

Becauſe though when ſeparated, they manifeſt a very powerful energy, by which they endeavour to unite; yet when united, they diſcover no perceptible action.

That the electric fluid ſhould be compoſed of two different elaſtic fluids or gaſes is agreeable to chemical analogy: thus atmoſpheric or common air is compoſed of reſpirable air, whoſe baſe is oxygen, and of mephitic air, whoſe baſe is azot; and though the particles of reſpirable air repel each other, as do thoſe of mephitic air, yet is there a ſtrong attraction between theſe two different kinds of air. See Lavoiſier's Elements, p. 84.

28. Excitation conſiſts in the decompoſition of the electric fluid.

The means by which this decompoſition is produced are various, being either ſimple contact, mixture, friction, evaporation, &c. The electric fluid, in experiments, is generally decompoſed by a cylinder and cuſhion, and againſt the cylinder is placed, on ſome non-conducting ſubſtance, a metallic cylinder, called the prime conductor: the cylinder by its cloſe contact with the cuſhion has its attraction for certain elements of the electric fluid increaſed, while at the ſame time its attraction for the other elements of it is diminiſhed;

diminiſhed; it extracts therefore from the cuſhion, while in contact with it, a portion of theſe elements; and communicates to the cuſhion an equal quantity of the elements of a different kind; but being quickly withdrawn from this contact, by the revolution of the machine, this ſuperabundant matter will lie diſengaged on the ſurface of the glaſs, and meeting with the points of the prime conductor will, from the nature of a conducting ſubſtance, quickly diffuſe itſelf over it; and at the ſame time the cylinder will attract from the prime conductor an equal quantity of the elements of a different kind, ſo as to recover its natural ſtate, and be ready for a freſh operation.

29. Non-conductors alone are capable of excitation.

Becauſe by not tranſmitting or tranſmitting very ſlowly the electric matter, they are capable of exhibiting ſigns of electricity.

30. If there be a conductor on one ſide of a thin electric plate, it cannot be excited.

Becauſe the ſuperabundant electric matter of one kind, expels from the other ſide, by means of the conductor, an equal quantity of the ſame kind, and attracts that of a different kind; and theſe different kinds of electric matter, ſtrongly attracting each other through the electric plate, become firmly attached to it.

On this principle, a glaſs veſſel, out of which the air had been exhauſted, on being rubbed ſhewed no ſigns of electricity on its external ſurface; becauſe the exhauſtion was probably imperfect; and therefore the attenuated air be-

came a conductor, See Art. 19. It has alſo been aſſerted, that if the air in the veſſel be condenſed, it cannot be excited; but this has been refuted by Beccaria, who has ſhewn, that when the condenſation is effected by the preſſure of mercury, there is a ready excitation; not ſo when it is effected by a piſton moiſtened with water or oil, which intermixes a conducting ſubſtance with the air. It is alſo to be obſerved, that a ſolid electric cylinder can be excited.

31. Non-conductors may on one part of their ſurface be excited, without diffuſing the ſame kind of electricity to the reſt of their ſubſtance.

32. If a conductor, inſulated and electrified, be touched by a perfect conductor, which communicates with the earth, it will loſe all its electricity at once; but an excited electric will not.

Becauſe the electricity belonging to the whole of the electrified conductor is eaſily conducted through its own ſubſtance, to that part which is touched by the other conductor.

Hence the electricity diſcharged from an electrified conductor is much more powerful than from an electric.

33. If a glaſs cylinder made very dry, be rubbed with a piece of ſilk, it will decompoſe the electric fluid, and accumulate on the prime conductor that electric gas which is called vitreous electricity.

This is otherwiſe called Poſitive electricity, according to the ſyſtem of Doctor Franklin: and the other gas is called

Reſinous

Resinous electricity; or Negative electricity according to Doctor Franklin's hypothesis.

34. If a piece of sealing wax be rubbed with a woollen cloth, its capacity for retaining the resinous electricity will be increased.

For if one pith-ball be electrified with excited glass, and another with excited wax, they will mutually attract each other; but if both be electrified with wax, or both with glass, they will repel each other; the electricity therefore of glass is different from that of wax. This difference was, at first, supposed to depend on the nature of the electric; and that one was the constant production of glass, and the other of wax, and such like substances. Hence the former was called Vitreous, and the latter Resinous electricity. But it was afterwards discovered, that each of these powers might be produced by the excitation either of glass or sealing wax.

35. Two bodies electrified positively repel each other.

This repulsion depends on the air, for in a vacuum there is no such repulsion.

The electric matter of an electrified body extends to a very small distance from the surface; and therefore in a vacuum must be insensible. But in the air, the electrified bodies have an intermediate substance to act on; and by disturbing the natural electricity of the particles of air which are nearest to them, and these the electricity of the next, and so on, the disturbance is propagated through all

the intermediate air, and thus the pith-balls appear to act at a sensible distance. But in this case, the electricity does not pass from either body into the air; and therefore after exhaustion, if the air be re-admitted, the repulsion *will* again be manifest.

36. Two bodies electrified negatively repel each other.

This has never yet been explained on the hypothesis of a single homogeneous fluid.

37. If one body be electrified negatively, and another positively, they will attract each other.

38. If one body be electrified positively or negatively, and another be in its natural state, *they* will neither attract nor repel each other.

For the superabundant positive electricity of the electrified body attracts the negative electricity of the body which is in its natural state, just as much as it repels the positive part. And the like argument holds when the body is electrified negatively.

This phænomenon appears inexplicable on the hypothesis of a single homogeneous fluid.

39. The species of electricity may be discovered by electric attraction and repulsion.

Hang on the electrified body two light pith-balls, suspended by thread; they will diverge with the electricity of the body; bring a piece of excited sealing wax gradually near them, if the balls separate further, on the approach of the wax, they are negatively electrified, or the electri-

city

city is of the ſame nature with that of wax: if, on the other hand, they come nearer together, the electricity is poſitive, or contrary to that of wax.

40. The hypotheſis of Doctor Franklin is, that the electric fluid is homogeneous; that a body is electrified poſitively or negatively, according as it has more or leſs than its natural portion of this fluid; and that all electrical phænomena ariſe from the reſtoration of this diſturbed equilibrium.

This hypotheſis ſeems to be inadequate; for, in the electric diſcharge, light and heat are both extricated from the electric matter, which cannot be accounted for by the ſimple current of an homogeneous fluid.

41. The hypotheſis of Mr. Eeles is, that there are two electric fluids, which have a ſtrong chemical affinity for each other, at the ſame time that the particles of each are ſtrongly repulſive of one another; and that all electrical phænomena ariſe from the ſeparation and re-union of theſe fluids.

This hypotheſis of Mr. Eeles appears to be well founded; but he ſeems to have conſidered merely a double current of electric matter, without any ſuppoſition of the chemical action of theſe different gaſes on each other. It is therefore equally inadequate with Dr. Franklin's to the explanation of the origin of the light and heat which are generated in the electric diſcharge; on which the principal phænomena in electricity depend. It is true that in Introd. page 40. he ſays that electric light and heat ariſe from the mutual

mutual condenſation of theſe two mediums; but, according to his hypotheſis, theſe mediums, when in their natural ſtate, are always in a ſtate of mutual condenſation, and therefore all bodies in their natural ſtate ought to appear luminous.

Of the Electric Charge.

42. If the electric gas of one ſpecies be increaſed on one ſide of a thin electric plate, it will be diminiſhed on the other ſide, if there be a conducting ſubſtance to convey it away; and the other electric gas will be increaſed.

Becauſe either electric gas is ſtrongly repulſive of itſelf, and attractive of the other; and the interpoſed plate is ſo thin as not entirely to obſtruct the electric energy.

The electric plate, in this caſe, is ſaid to be charged.

43. If the electric gas at one ſide of an electric plate be diminiſhed, the gas of the ſame kind at the other ſide will be increaſed.

44. The force with which the contrary gaſes attract each other, is leſs than the attractive force with which their particles are held combined with the particles of the electric.

For the component particles of all bodies are ſuppoſed to be combined with electric particles; now if the attraction between the contrary gaſes on the oppoſite ſurfaces were either equal to or greater than the force with which the

the component electric particles are held by those of the electric, it would become a conductor.

45. In the charge, the quantity of electricity of the electric is always very nearly the same with that of the natural state.

Because as much as either gas is increased on one side of the electric plate, so much very nearly will it be diminished on the other.

46. The nearer the surfaces of a charged plate are to each other, that is, the thinner the plate, the greater is the force of the charge.

Because the action of each gas is less obstructed by the intervention of the electric.

47. An electric plate may be so thick as to be incapable of a charge.

This will happen when the particles of the gas on one side of the plate are removed to such a distance, as not to be able to act on the gas at the other.

48. Glass, though not absolutely impervious to the electric fluid, is nevertheless so bad a conductor as to be capable of a charge.

49. An electric plate may have its quantity of electric matter on one surface increased, in a certain degree, without any diminution on the opposite surface.

Thus if a phial be insulated, and its knob connected with the prime conductor, and the machine put in motion, a certain quantity of electric matter will be added to the inside;

side; for if you touch the outside, a quantity nearly equal to that thrown in, comes from it. Previous to the action of the machine, the two surfaces are in equilibrium, but the action endeavours to condense and accumulate the electric matter on the inside with a force equal to that with which it accumulates it on the prime conductor; and this increased force on the inside will accumulate the matter there, untill the increased repulsive force arising from that accumulation or condensation is equal to the accumulating force.

When the phial is touched, the outside becomes negative, and the vial will be charged in a low degree.

It is however to be remarked, that some electricians assert, that the reason why the matter can be *accumulated* at the inside of an insulated phial, is because the air is not a perfect conductor. But were this so, the phial would be charged, previous to touching the outside.

50. A plate of glass may be positively electrified at both sides.

Cavallo and Morgan assert the contrary; and that when a glass plate appears positive on both sides, it is caused by the superior positive power on one side, which predominates over the weaker negative of the other. But even thus they relinquish their principle, that no electric matter can be accumulated on one side, unless an equal quantity passes off at the other. Mr. Eeles on the other hand asserts, that both sides can be electrified positively, as may be incontestably proved by touching either side with an electrometer, which will thus acquire positive and permanent electricity, whichever of the sides it touches.

51. An

51. An insulated electric cannot be charged.

Although the electric gas of one species may be increased on one side of the electric, yet the other side being insulated, must continue in its natural state, and therefore there will be no charge.

52. The charge of an electric plate resides in the electric, and not in the coating.

Of the Electric Circuit.

53. If a communication be suddenly made between the positive and negative sides of a charged electric, the equilibrium will be suddenly restored.

This is called completing the circuit; and the act of union of the contrary electric gases is called the electric shock.

54. The equilibrium is not perfectly restored by once completing the circuit.

For the superabundant electric matter, which has spread from the metallic coating upon the surface of the non-electric, takes some time to return into that coating, after the part in contact with the coating has been restored to its natural state by the discharge.

55. A strong electric shock, whatever may be the length of the circuit, appears to be performed in an instant, through a good conductor.

A weak shock has been found to take some little time through a long and imperfect conductor.

56. The very same individual electric gas which

caufes the fuperabundance on the pofitive fide, is not transferred to the negative fide, when the circuit is completed.

For the contrary electric gas of the conductor which forms the circuit, is firft attracted by the negative fide.

57. If an electric plate be charged negatively, and the circuit completed, the conductor which forms the circuit will be negatively electrified after the difcharge.

Becaufe the accumulated electric gas at one fide of the electric is fo far obftructed by the intervention of the plate, as to expel a lefs quantity on the other fide.

58. If the plate be charged pofitively, the conductor, after the difcharge, will be pofitively electrified.

59. If there be two different conductors to complete the circuit, the difcharge will always be made through that which conducts beft, every thing elfe being the fame.

60. If the conductors be equally good, but one fhorter than the other, the difcharge will be made through the fhorteft.

61. If the conductors be equally good, and of the fame length, the difcharge will be made equally through them all.

The force of the difcharge, in a given circuit, may be thus weakened at pleafure.

62. The

62. The force of the electric ſhock is weakened by the length of the circuit.

63. If any part of the human body form a portion of the electric circuit, it is obſerved, that the diſcharge is perceived in that part only which forms the communication, unleſs the charged ſurfaces be very great.

64. If a conductor be connected with one ſide of a charged plate, though it does not make part of the circuit, part of the diſcharge will paſs through it.

This is called the lateral exploſion; and is cauſed by the interruption in the circuit, made by introducing into it bad conductors, or ſuch as are too ſmall.

65. Electricity finds ſome obſtruction in paſſing through even the beſt conductors.

For in ſome caſes it will prefer a ſhort paſſage through the air, to a long one through the moſt perfect conductors.

66. The electric ſhock diſplaces the air through which it paſſes.

This is proved experimentally by Kinnerſley's air electrometer.

67. If its paſſage from conductor to conductor be interrupted by non-conductors, of a moderate thickneſs, it will rend and tear them in pieces.

68. If one ſurface of a charged plate be perfectly inſulated, although the other communicates

with the earth, no diſcharge of either ſurface will follow.

For the ſuperabundant electric matter of either kind, accumulated on the inſulated ſide, cannot eſcape, it will therefore, by its repulſive force, preſerve the other ſurface in a contrary ſtate.

69. If two electric plates be charged, and a communication be formed between the poſitive ſide of one, and the negative of the other, no diſcharge will follow, unleſs a communication be formed between the other two ſides at the ſame time.

Becauſe the negative ſide of one plate attracts the particles on the poſitive ſide with a force equal to or greater than the force of the negative particles of the other plate.

Of the Electricity of the air.

70. A plate of air may be charged, as any other electric.

71. The ſtrength of the excitation being the ſame, a charged plate of air will be more eaſily broken, the thinner the plate.

72. The ſtrength of the excitation being the ſame, a charged plate of air will be more eaſily broken, cæteris paribus, the leſs its denſity.

73. The electric powers poſſeſſing the oppoſite ſurfaces of a cylindrical plate of air, whoſe diameter is evaneſcent, will be incomparably leſs re-

ſiſted

ſiſted by the interpoſed electric, than by the thinneſt plate of air contained between ſurfaces of a finite magnitude.

For the leaſt cylinder, whoſe diameter is finite and thickneſs evaneſcent, is incomparably greater than the leaſt cylinder whoſe altitude is finite, and diameter evaneſcent. See Atwood's Electricity.

74. Points ſilently diſcharge the electric fluid.

When a pointed body is preſented to any electrified ſurface, a cylindrical plate of air, of evaneſcent diameter is charged with the contrary electricities, which therefore attract each other through the interpoſed electric; and to this attraction there will be little reſiſtance, becauſe the quantity of interpoſed air is very ſmall.

75. If a pointed conductor be applied to any diſengaged electric matter, it will quickly withdraw it, and communicate it to the ſurface of the ſurrounding air.

For this reaſon it is, that the prime conductor is terminated with points, which are preſented to the excited electric, and which therefore draw off the electric matter, which by the friction has been diſengaged from the electric, and lies looſe and uncombined upon its ſurface.

76. The electric fluid appears a diverging ſtream from a point electrified poſitively; and like a ſmall ſtar on a point electrified negatively.

Mr. Morgan has ſhewn, when the electric diſcharge is made through a fluid medium, which is not a good conductor,

conductor, that the flaſh proceeds luminouſly and with ramifications from one end of the interval to the other. If the conducting power of the medium be increaſed, the light will paſs in a continued, undivided ſtream; and if the interval be then ſufficiently increaſed, the light will be viſible only at the extremities of the paſſage; in conſequence of the greater condenſation of the light at theſe extremities. See Lectures on Elect. vol. 2. p. 98, 99. It is farther obſerved, that if there be two bodies, placed near each other, which are diſſimilarly electrified, the poſitive electricity paſſes in a divided, ramified flaſh; and the negative in one continued, undivided line of light. See Mr. Nicholſon's curious experiments in Adams's Elect. p. 222. We may therefore infer, from comparing theſe experiments, that the air is a better conductor of negative than of poſitive electricity; and probably it is to this difference that we ſhould attribute the different appearances of the ſtar and pencil. Poſitive electricity paſſes out from the electrified point into a medium which does not conduct it well; the light is therefore viſible on all ſides to a ſenſible diſtance, and conſequently appears as a luminous cone or pencil of light. But the negative electricity, paſſing into a medium which conducts it better, is viſible only at that extremity where there is a greater conſtipation of the electric light. This conjecture ſeems to be confirmed by the following experiment, which was more than once made with particular care, and deſerves to be repeated: two equal and ſimilar prime conductors, each furniſhed with a pith-ball electrometer, were equally electrified at the ſame time,

one

one poſitively, the other negatively; it was found, that the poſitive conductor retained its electricity much longer than the negative conductor.

77. A current of air blown againſt a ſtream of electric matter does not affect its appearance.

78. There is a ſenſible current of air at an electrified point, which is always in the direction of the point, whether the electricity be poſitive or negative.

For the air contiguous to the point becomes poſſeſſed of the ſame electricity, this plate of air therefore and the point repel each other; conſequently, in both caſes, the air will move from the point.

79. The electricity of an electrified body does not diſplace the ambient air.

For the force with which the body attracts the electric matter extends to an indefinitely little diſtance.

80. The atmoſphere is always electrified, but moſt commonly with poſitive electricity.

81. Aerial vapours are endued with electricy.

82. Hail is always attended with electricity.

Of Influential electricity.

83. An electrified body brought within a certain diſtance from another, which is inſulated and in its natural ſtate, will produce the ſame kind of electricity

electricity in the remote end of the body, which is not electrified; and the contrary electricity in the nearer end.

Becaufe the fuperabundant gas of the electrified body attracts the heterogeneous gas, and repels that which is homogeneous, in the near end of the body not electrified.

84. If the infulated body be of confiderable length, the contrary electricities will follow each other in alternate fucceffion.

85. If an electrified body be brought within a certain diftance from another which is in its natural ftate, a fpark will pafs between them.

This is called the ftriking diftance. The electrified body firft produces a contrary electricity in the other, the intervening plate of air is thus charged, and when the thicknefs of this plate of air, or the diftance of the bodies is fufficiently fmall, a difcharge will take place, which is manifefted by a flafh or fpark.

86. The electric fpark will pafs through a greater interval of air to a conductor, the more perfect its conducting power.

Becaufe the more perfect it is, the more perfectly will the electrified body induce a contrary electricity in the part which is next it; and therefore the greater will be the tendency of the two electricities to unite.

87. Long fparks are always inflected in various directions.

This

This is caused by the electric matter passing through those parts of the air, in which the best conductors are found.

88. If the body which was previously in its natural state be insulated, after the spark passes, it will be found to have acquired the same electricity with the electrified body.

For part of the superabundant gas of the electrified body will pass from it, and thus both bodies will be electrified in the same manner.

89. If two insulated conductors be just within the striking distance of each other, and an electrified body approach one of them, a spark will pass between the conductors; that which is more remote from the electrified body assuming the same kind of electricity, and that which is nearest to it, the contrary: if the electrified body be then removed, the spark will return, and both conductors will resume their natural state.

This is called the electrical Returning Stroke. See Lord Mahon's Electricity.

90. An excited electric, after its power has been so reduced, as to render it incapable of communicating its proper electricity, does however still retain a permanent degree of that power, by which it can produce a small degree of influential electricity in a conducting body which is in contact with it.

91. If a conducting fubftance be brought into contact with an excited electric, and touched with a conducting fubftance, and then removed by an infulating handle, it will poffefs an electricity contrary to that of the excited furface.

For if the excited furface be pofitively excited, when the conducting fubftance is brought into contact with it, and at the fame time touched by a conductor communicating with the earth, all the communicated fuperabundant electric matter will firft pafs off; and the equilibrium of the remaining natural quantity will be difturbed, the part next the glafs being negative, and the remote part pofitive; but the contact of the conductor ftill continuing, the accumulated part of the remote furface will likewife pafs off, fo that when the conductor is removed, and the conducting furface taken up by a non-conducting handle, it will be negatively electrified. The contrary will happen, if the excited furface be negative. See Milner's Electricity.

92. If two bodies, equally and contrarily electrified, be applied at the fame time to a third body, they will counteract each others effects.

93. If they be electrified unequally, the ftronger will manifeft its proper effect, in proportion to the difference of the powers.

94. If one fide of an infulated coated plate communicate with a body negatively electrified, that fide will become negative, and the other fide will either continue in its natural ftate, or become

ſlightly poſitive; nevertheleſs an electrometer applied to this latter ſide, will diverge with negative electricity.

In this caſe the negative electricity, by its ſuperior force, influences the electrometer through the interpoſed plate.

95. If the poſitive ſide of an electric plate or jar, ſlightly charged, be applied to a body ſtrongly electrified with poſitive electricity, an electrometer applied to the negative ſide, will diverge with poſitive electricity.

For when two electricities are applied at the ſame inſtant to the ſame body, the effect is produced by their difference: in the preſent caſe, the poſitive electricity predominates over the negative, even through the interpoſed electric plate.

MAGNETISM.

Of Magnetifm in General.

1. MAGNETISM is that power, by which the load-ftone attracts iron, and other concomitant effects are produced.

The natural magnet or load-ftone is an iron ore, which contains a greater quantity of iron, either in the metallic ftate, or not much oxygenated, than moft other iron ores. It alfo often contains a portion of quartz and argill, and probably fome fulphur, becaufe, when made red hot, it has generally a fulphureous fmell. It is about feven times heavier

heavier than diftilled water; is of a dull brownifh black colour; its hardnefs is fuch as juft to afford fparks, when ftruck with fteel; it is found almoft wherever there is a good iron mine; but not of any particular fhape or fize.

2. The attraction between the magnet and iron is mutual.

3. The attraction between the magnet and iron is fubject to a variation, there being a limit to the weight and fhape of the iron, in which it will be attracted moft forcibly; which limit can be determined only by experiment.

4. Magnetifm fubfifts between the magnet and iron only.

Platina has been by fome fuppofed to be magnetical, but more accurate experiments fhew that it is not. One kind of Bifmuth is faid to be repelled by the magnet in all cafes. See Cavallo, page 71. Almoft every kind of animal and vegetable fubftance is affected by the magnet, after being burned, but not before. Colourlefs precious ftones, as the diamond and cryftals, are not attracted; neither the amethyft, chalcedony, or topaz, or fuch as lofe their colour by fire; but all others, as the ruby, chryfolite, and tourmalin are attracted. The emerald, and particularly the garnet, are not only attracted, but frequently acquire an evident polarity. All fuch action we muft afcribe to particles of iron in the compofition of thefe bodies.

5. If a needle be brought near a magnet, there will be found two points on the furface of the magnet,

net, towards which the needle will ſtand perpendicularly.

Theſe points are called the poles, and the line which joins them, the axis of the magnet.

6. The ſame magnet has frequently more than two poles.

This is to be attributed to the heterogeneous nature of the magnet.

7. Homogeneous poles repel, and heterogeneous poles attract each other.

For the two poles of two magnets which are attracted by the ſame pole of a third magnet, are homogeneous poles; but theſe are found to repel each other.

8. There muſt be ſome point between the poles of a magnet, at which their forces are equal.

This point is called the magnetic centre; but it is not always in the centre of the magnet.

9. If a magnet be broke into two fragments in a tranſverſe direction, a magnetic centre will ariſe in each fragment, which at firſt will be nearer to the fractured end, but in time approaches towards the middle of the fragment.

Since the particles of the magnet are within the ſphere of each others action, they will act with more advantage on thoſe which are in the middle of the magnet, than on ſuch as are at any diſtance from it; and conſequently the central particles will be more orderly diſpoſed according to their

their polarity. Therefore when the magnet is broke, the end next the fracture will be more powerful than the other.

10. The force of each portion of the magnet separately considered may be conceived to be concentrated in some point between the extremity of the magnet and the magnetic centre.

11. If a magnet be cut through its axis, the parts which were before in contact, will now repel each other.

Because they are homogeneous poles by Art. 7.

12. If a magnet be cut in a direction perpendicular to the axis, each part will be a perfect magnet, the polarity of the parts which were before in contact being contrary.

13. If a magnet be pulverised, every particle will be found to be a complete magnet, endued with two contrary poles.

14. If a magnet be pulverised, and the particles then put into a glass vessel, the magnetic virtue will either vanish, or will be found to be very much weakened.

Because the position of many of them will be inverted, and will therefore counteract each other, one attracting what the other repels.

15. The component particles of a magnet being all complete magnets in themselves, some philosophers have supposed, that in its original formation,

they

they would arrange themselves in an orderly position, the homogeneous poles looking the same way, so that the heterogeneous poles, which attract each other, would be in contact.

Clare, in his Motion of Fluids, adopts this opinion from Sir Isaac Newton's Queries: "It is not improbable," says Newton, "but that there may be a polarity in many other parts of matter, as well as in the magnet and iron, in which they are certain and incontestable. That there is such a property in several fixed and crystallised salts, is pretty apparent, by their always ranking and disposing themselves in one certain, unalterable manner, as often as they are reduced from a fluid to a fixed state." But this opinion is more particularly insisted on and illustrated by Mr. *Kirwan*, in the Trans. of the R. I. Acad. vol. 6.

16. The smallest quantity of antimony mixed with iron destroys its polarity.

This probably arises from the magnets crystallising in a manner different from that which takes place in the *natural* magnet; for different substances assume different forms in crystallising.

17. Extreme cold has an influence on the magnet.

Captain Ellis, in his Voyage to Hudson's Bay, relates, that in an intense cold, his compasses lost their directive power; and that on bringing them near the fire, they soon recovered it. This seems analogous to the preceding article.

18. Heat weakens the power of a magnet.

For,

For, by the heat, the particles are thrown into a tremulous motion, so that the regular disposition of the homogeneous poles is disturbed.

19. The attractive force of the magnet varies in the inverse duplicate ratio of the distance

This is proved by the experiments of Monf. Coulomb, Acad. Science, Paris, 1785. Former philosophers failed in their researches as to this law, because they did not take into their calculation four distances, as is necessary, viz. between the centre of action of each of the two poles of the attracting and attracted body. The same thing is likewise proved by M. Lambert, Mem. Berlin, Tom. 22; but, I believe, first of all by Mr. Michell. Muschenbroek thought that the repulsive power, at equal distances, was less than the attractive; but Mr. Michell says, that this is a mistake; which arose from his not considering, that when the contrary poles are placed together, the power of the magnet is increased; but if the synonymous poles be placed together, its force will be diminished.

20. The attractive force of similar magnets is proportional to their surfaces.

This appears from the experiments of M. Dan. Bernouilli. See Journ. Helvetiques, Nov. 1758; and also a letter to Monf. J. Trembley, inserted in Saussure's Journey to the Alps, vol. i. p. 58. See also Hutton's Magnetism, p. 72. This fact seems to suggest the operation of a magnetic fluid, which acts, like the electric fluid, in proportion to the surface, and not as gravity, in proportion to the quantity of matter.

21. Smaller natural magnets have a greater attractive power than larger, in proportion to their ſize.

This follows from the laſt article.

22. A natural magnet, cut off from a larger, will ſometimes be more powerful than the entire original load-ſtone.

This muſt be attributed to the heterogeneous matter in the larger one, which obſtructs the action of the purer part.

23. A plate of iron, but no other body, *interpoſed*, can impede the operation of the load-ſtone.

Of the Influence of Magnets on each other.

24. If a magnet be ſuſpended near another which is fixed, the former will be turned out of its uſual poſition, ſo that its axis ſhall approach towards coincidence with the axis of the fixed magnet.

This is called the directive power of the magnet; and follows from Art. 7.

25. The directive power of a magnet extends to a greater diſtance than its attractive power.

For the former ariſes both from the attraction of the heterogeneous and repulſion of the homogeneous poles; the latter ſubſiſts only between heterogeneous poles.

26. The magnetic power in two magnets will be increaſed, by letting them remain with their contrary poles together.

27. If

27. If the homogeneous poles of two magnets be placed together, they will deſtroy each other's effects.

28. If the homogeneous poles of a ſtrong and a weak magnet be placed together, the polarity of the weaker will be firſt diminiſhed, then annihilated, and at laſt reverſed.

29. If the homogeneous poles of a ſtrong and a weak magnet be placed oppoſite and near to each other, the magnetic centre of the weaker will gradually recede towards the extremity which is fartheſt from the ſtronger magnet, until at length, when the poles are reverſed, a new magnetic centre will ariſe.

For the magnet will firſt change the poſition of the particles in the end next to it, to ſome certain diſtance; and the particles which retain their original poſition, for a certain portion, will form a counteracting power to that of the inverted particles; therefore the magnetic centre will be nearly in the middle of the remaining part of the magnet, and conſequently nearer to the remote extremity of the bar.

30. A magnet attracts another of equal ſtrength more forcibly than a piece of iron of any dimenſions.

See Van Swinden's Mem. vol. ii. p. 500.

31. A magnet attracts a weaker magnet more

 forcibly

forcibly than a piece of iron of the ſame hardneſs and dimenſions.

32. If a ſmall piece of iron be brought into contact with a magnet, and adhere to it; and if a bar of iron, not magnetic, be then brought near to the load-ſtone, but without touching it, the ſmall piece of iron, as ſoon as it is touched by the bar, will follow it, and deſert the load-ſtone.

See Mem. de l'Acad. 1777, p. 276. 283 and Æpinus Tent. §. 160.

Of Artificial Magnets.

33. If a bar of iron be brought near a *magnet, it* will be rendered magnetical, that part of it which is neareſt to the magnet acquiring a contrary polarity.

Iron rendered magnetical is called an artificial magnet.

34. In whatever manner an oblong piece of iron be applied to the load-ſtone, it will receive its virtue only lengthways.

35. If an artificial magnet be cut through the axis, the parts which were before in contact will repel each other.

This is conformable to Art. 11.

36. If an artificial magnet be cut in a direction perpendicular to the axis, each part will be a perfect magnet, the polarity of the parts, which were before in contact, being contrary.

This is conformable to Art. 12.

37. Since

37. Since an iron bar rendered magnetic, has all the properties of a natural magnet, we may conclude, that its conſtituent particles are endued with polarity, and are brought into a regular diſpoſition by the action of the magnet.

38. If a ſmall dry glaſs tube, filled with iron filings, preſſed cloſe, be touched with a magnet, as if it were an iron bar, the tube will be rendered magnetical; and if the tube be then ſhaken, ſo that the ſituation of the filings may be diſturbed, the magnetic virtue will vaniſh.

Becauſe all the particles are made, by the touch, ſo many magnets, whoſe homogeneous poles look the ſame way; but when ſhaken, this orderly diſpoſition is diſturbed.

39. A bar of ſoft iron acquires and loſes magnetiſm ſooner than a hard bar.

40. If a bar of iron be preſented to the magnetic centre, it will not acquire magnetiſm.

41. If a ſmall magnetic needle moveable on a pivot; be placed between two magnets, ſo that the heterogeneous poles of the needle and the magnets may be near each other, the needle will be thrown into a violent vibratory motion, which will continue for ſome time, and at length the needle will become perfectly quieſcent.

42. A long bar of iron or ſteel, or a long piece of iron, ſuppoſe of three or four feet, may have ſeveral poles following each other alternately.

For

For the pole of the magnet next the iron bar generates a contrary polarity in a certain portion of it; and that part, for the same reason, generates a contrary polarity in the next part, and so on. These successive poles become gradually weaker in power, according as they recede from that end of the bar, which is contiguous to the magnet.

This consecution of poles resembles the alternate succession of the contrary electricities in an insulated conductor, See Art. 82. Electricity; and therefore suggests the existence of a magnetic fluid.

43. When several poles alternately follow each other, as in the last article, the first magnetic centre is very near that end of the bar, which is next to the magnet.

44. A magnet cannot communicate its action through an iron bar which exceeds a certain length.

This length depends on the force of the magnet; but it very seldom exceeds six feet.

45. There is no magnetic attraction but between the contrary poles of two magnets.

For iron presented to a magnet must itself become a magnet before it will be attracted.

46. Both poles of a magnet equally attract needles till they are touched; then it is, and then only, that one pole begins permanently to attract one end, and to repel the other.

47. To find whether a bar of iron or steel be magnetic, apply one end to a magnetic needle, if it

it attracts both ends of the needle, it is not magnetic; but if it attracts one end, and repels the other, it is magnetic.

48. If one end of a magnet be drawn along a needle, or any bar of iron or ſteel, ſeveral times in the ſame direction, the needle or bar will become magnetic; and that extremity of the iron or ſteel, which the magnet touched laſt, acquires a polarity contrary to that of the end of the magnet which touched it.

This is called the ſingle touch, and is conformable to Art. 33.

49. If the magnet be drawn in a contrary direction, it takes away by the ſecond ſtroke, the magnetiſm which it communicated by the firſt.

50 If two magnetic bars, parallel to each other, and whoſe homogeneous poles lie contrary ways, be ſet perpendicularly on an horizontal bar, and then rubbed from the middle to one end; and ſo backwards and forwards, ſeveral times, ending at the middle, the horizontal bar will be rendered magnetical; and its ends will acquire a polarity contrary to that of the reſpective ends of the perpendicular magnets which touched them.

This is called the double touch, and is founded on the ſame principle with Art. 48.

51. The

51. The power of a magnet is rather increaſed than diminiſhed by communicating magnetiſm to other ſubſtances.

52. A magnet cannot communicate a greater power than itſelf poſſeſſes.

53. The attractive power of a magnet may be increaſed conſiderably, by gradually increaſing the weight which it ſuſtains.

54. The holding a piece of iron to one *pole of* a magnet, increaſes the power of the other.

This follows from Art. 51.

55. Several magnets of nearly an equal degree of magnetiſm, when joined together, *have a ſtronger* power than one of them ſingly.

56. If two pieces of ſoft iron be adapted to the two poles of a magnet, ſo as to project on the ſame ſide of the magnet in a direction perpendicular to its axis, theſe pieces themſelves being rendered magnetical, another piece of iron may be applied to their projections, ſo as that both poles may act in conjunction.

To determine the quantity of iron which is to be thus applied, try the magnet with ſeveral iron bars, and the greateſt weight which it takes up, will be the weight required.

This

This is called the armature. For the ſame purpoſe, and to avoid the armature, magnets are often made in the form of a horſe-ſhoe, having the poles in the extremities.

57. The magnetiſm of iron is ſtronger in proportion as it is more diſoxygenated.

58. There are ſome refractory mines of iron, which are not magnetical till after calcination.

59. Vitrification does not change the magnetiſm of iron.

60. Pulveriſation makes no change in the magnetiſm of iron.

Of the Magnetiſm of the Earth.

61. The earth has been ſuppoſed to be a great magnet.

For it produces all the phænomena of a natural magnet.

The learned Mr. Kirwan has with great ingenuity endeavoured to ſhew, that the earth is a great magnet, formed by the chryſtalliſation of its iron and magnetic ores, from a fluid ſtate, in which it exiſted at its formation. And as the ſhoots of the chryſtalls would be found in that direction in which they were leaſt diſturbed, they would all lie orderly in a direction parallel to the axis of the earth, and the axis of the magnet coincide with the axis of rotation.

Neverthelefs, there appear ftrong objections to this hypothefis, See Æpinus, p. 300: As 1. Iron is not found to be heavier near the poles, than towards the equator. 2. If a bar of foft iron be held vertical, it is rendered magnetic; if horizontal, it quickly lofes that power; therefore in the vertical pofition it ought to be heavier than in the horizontal, which does not appear to be the cafe, See Cavallo, p. 93. 3. If a magnetic needle, lying on a piece of cork, be floated on water, it ought always to move to the northern fide of the veffel, and not continue at *reft* in the middle. 4. Though many ftones and ores are impregnated with iron, yet they are not in that ftate magnetical, See Cavallo, p. 16. that is, they will not affect a magnetic needle, though they themfelves *may be* affected by a magnet. 5. The magnetifm of the earth feems incompatible with the variation of the needle.

62. The poles of any magnet will be directed one towards one of the magnetical poles of the earth, the other towards the other.

This follows from art. 7 and 61. The pole which is directed towards the north pole of the world, is called the north pole of the magnet; the other the fouth pole.

63. The magnetic axis of the earth is not coincident with the axis of revolution.

For the north pole of the magnet is not directed exactly towards the pole of the world. This is called the variation.

64. The magnetic needle is fubject to a diurnal variation, moving in northern latitudes, generally towards

towards the weſt before noon, and afterwards gradually returning.

It has been conjectured that this may ariſe from the diurnal change in the heat of the earth; for the eaſtern parts of the earth being heated faſter in the morning than the weſtern, their attractive force on the needle will be weakened, by Art. 18, and therefore the needle will move weſtward.

But the magnetical nucleus, to which we attribute the direction of the magnetic needle, is certainly buried at a very conſiderable depth below the ſurface of the earth; whereas we know, that the line which ſeparates the terreſtrial cruſt, ſubject to the influence of heat and cold, from that which is not ſubject to it, does not lie far below the ſurface, for in caves of even moderate depth, the thermometer preſerves a permanent ſtate.

65. The magnetic needle is ſubject to an annual variation.

Doctor Halley endeavoured to account for this phænomenon, by ſuppoſing, that the axis of the magnetic nucleus was not exactly coincident with the axis of the earth, and that this nucleus was alſo moveable within the body of the earth. However it has been found, that the variation is not regular in any place, as it ought to be on this hypotheſis. It is however ſingular, that the Ætites, or Eagle ſtone, which is of the claſs of iron ores, contains a nucleus, which is frequently moveable in the centre of the ſtone. See Fourcroy's Chem. V. 3. p. 219.

 66. The

66. The Aurora Borealis produces irregularities in the declination of the needle.

For during an aurora borealis the needle is, in general, much difturbed, while a fimilar needle of brafs is not at all agitated. Since it is unqueftionable that the aurora borealis has a magnetical influence on the needle, and that the needle is fubject to a daily variation, perhaps the aurora borealis exifts daily in the atmofphere, fo as to produce this daily variation, and the annual variation alfo.

67. If a magnet be fufpended on an horizontal axis at its centre of gravity, fo that it may vibrate in a vertical circle, the north pole of the magnet, on the northern fide of the equator, will be *depreffed*; and the fouthern pole depreffed in the *fouthern* latitudes.

Becaufe in Northern latitudes, the influence of the northern magnetic pole of the earth is predominant. This is called the dipping needle. At the equator there is no dip. If a needle were placed exactly E. and W. at the equator, it would remain fo; but on the flighteft agitation it would traverfe, and point N. and S.

68. If a bar of foft iron be kept vertical, or rather parallel to the magnetic axis of the earth for fome time, it will become magnetic, the lower end acquiring a north polarity in the northern latitudes; but in the fouthern parts of the earth it acquires a fouth polarity. On reverfing the bar, the poles are immediately

immediately changed. This follows from Art. 33 and 61.

69. In northern latitudes, the ſouth pole of a magnet is ſtronger than the north pole

70. If an iron bar made red hot, be left to cool in the magnetic line, it will acquire a degree of magnetiſm, which is more or leſs permanent according to the nature of the iron.

For the iron, while red hot, is ſoft, and therefore the earth, or perhaps ſome atmoſpherical cauſe, can more eaſily render it magnetical; but when cooled, it becomes harder, and conſequently more tenacious of the power it has acquired.

71. If an iron bar held vertical, be rubbed always in the ſame direction againſt an horizontal bar from one end to the other, the horizontal bar will become magnetical, that extremity which was firſt touched, being the north pole in the northern parts of the earth.

For the vertical bar either by the action of the earth, or of ſome cauſe exiſting in the atmoſphere, becomes a magnet, whoſe lower end therefore acquires a north polarity by Art. 68. therefore by Art. 48. the extremity laſt touched acquires a contrary polarity, that is, a ſouthern polarity; and of courſe, the extremity which is firſt touched, acquires a northern polarity.

72. If an horizontal bar be rubbed from both ends

ends to the middle, it will have two north *poles*, one at each end, and a fouth pole in the middle.

73. If the horizontal bar be rubbed both ways from the middle to the two extremities, *it* will have two fouth poles, one at each end, and a north pole in the middle.

74. If an iron bar be held vertical, a few fmart ftrokes of a hammer will give it polarity.

This fhews, that a certain difpofition of the particles of iron is requifite, in order that it fhould be magnetical; which is the opinion of Van Swinden. See his Memoirs, Vol. 1. p. 479.

75. If a bar, weakly magnetifed, be *held vertical*, and ftruck alternately at each end, its polarity may be deftroyed or reverfed.

If the polarity be deftroyed, we may conclude from Art. 38, that the homogeneous poles of the component or elementary magnets, are thrown into contrary pofitions, by the contrary vibrations produced by the ftrokes of the hammer at each end of the bar. If it be reverfed, by parity of reafon we infer, that the greater part of the particles have the pofition of their poles inverted.

76. The electric fhock frequently gives polarity to iron bars through which it is tranfmitted.

For in its paffage through the bar, it agitates the particles of the iron, and therefore produces an effect fimilar to that in Art. 74. So that electricity, as fuch, does not contribute to the communication or deftruction of the magnetic

netic virtue, but merely on the principle of exciting a tremulous motion amongſt its particles, ſo as that the earth or atmoſphere may give that diſpoſition, on which polarity ſeems to depend.

77. The aurora borealis is probably a magnetic meteor.

For 1. the northern pole of the needle appears to follow the aurora borealis, See Van Swinden's Mem. Vol. p. 247. 2. The rays of the aurora borealis ſeem to converge to the magnetic pole. See Mairan, and Encyclop. Brit. alſo Cavallo, p. 331, and Meteorological Obſervations and Eſſays by J. Dalton, An. 1793. 3. A magnetic needle appears much diſturbed during an aurora borealis, while a ſimilar needle of braſs is not agitated.

Of the Cauſe of Magnetiſm.

78. There is no direct experiment by which the exiſtence of a magnetic fluid can be proved.

The opinion that magnetiſm was occaſioned by a fluid, entering in at one pole, and paſſing out at the other, took its riſe from the following experiment: having put a ſmall magnet among ſome iron filings, laid upon a piece of paper, give the table a few gentle knocks with your hand, ſo as to ſhake the filings a little, and they will diſpoſe themſelves in curves terminating at the poles, and concave towards the axis of the magnet. But this effect is occaſioned merely by the action of the magnet on the filings, each particle becoming

becoming itſelf a magnet; ſo that there are formed ſeveral ſtrings of magnets, reaching from one pole of the central and principal magnet to the other.

79. Neverthelefs, it ſeems that the *exiſtence of* a magnetic fluid muſt be admitted; becauſe we cannot conceive a body to act where it is not.

"That gravity," ſays Sir I. Newton, "ſhould be innate, inherent and eſſential to matter, ſo that one body may act upon another at a diſtance, through a vacuum, without the mediation of any thing elſe, by and *through* which their action and force may be conveyed from one to another, is to me ſo great an abſurdity, that I believe no man, who has in philoſophical matters a competent faculty of thinking, can ever fall into it." See *Biſhop Horſeley's* Newton, vol. iv. p. 438.

80. It ſeems probable that magnetic phænomena ariſe from cauſes exiſting in the atmoſphere.

The magnetic needle is certainly affected by atmoſpherical cauſes; and therefore all its phænomena, perhaps, depend on the ſame cauſes. The magnetiſm of the *earth* is an hypotheſis, but the influence of atmoſpherical cauſes on the needle is a fact.

81. Magnetic phænomena depend on a medium more ſubtle than air.

For the magnet attracts iron with the ſame force in vacuo as in the open air.

82. Electricity and magnetiſm do not interrupt each other's operation.

For

For an electrified magnet attracts light bodies of all kinds by its electric power; at the same time that it attracts iron by its magnetic virtue.

83. The causes of electricity and magnetism are different.

Because, 1. Electricity acts on all bodies, magnetism on iron only. 2. Electricity affects the senses, magnetism does not. 3. Points neither supply nor absorb the magnetic fluid more abundantly than blunt bodies, as they do in electricity. 4. Moisture diminishes electrical action, but has no influence on magnetism. 5. The whole of any substance may acquire one kind of electricity throughout; but every magnetic body has both kinds of magnetism. 6. The Aurora Borealis is not an electrical meteor, yet it influences the magnetic needle.

84. Though the electric and magnetic powers are different, yet there subsists a strong analogy between them.

As, 1. Electricity is of two kinds; so is magnetism. 2. Bodies similarly electrified, or similarly magnetised, repel each other; if dissimilarly, they attract each other. 3. There is no electrical or magnetical attraction except between bodies differently electrified or magnetised. 4. If a body be brought near another which is electrified, its end next the electrified body acquires the contrary electricity, and the remote end of it the same kind of electricity; so in magnetism; and in this case the neutral point is analogous to the magnetic centre. 5. The different kinds of electricity and magnetism sometimes succeed each other alternately, for several times in the same body; so also in magnetism. 6. One kind of electricity or magnetism cannot be produced without

the other. 7. A body more powerfully electrified or magnetised than another which is in the contrary state, when applied to it, will change its electricity or magnetism. 8. If an excited electric be broke transversely, the parts which were before in contact will be found diversely electrified; so in magnetism. 9. The electric and magnetic powers are proportional to the surfaces, not to the solid contents of the electrified and magnetic bodies. 10. A considerable degree of heat destroys both electricity and magnetism.

85. Animal magnetism appears to be a mere figment; and all the effects ascribed to it have been produced either by the imagination, or by drugs secretly applied.

86. Medical effects have been produced on the human body by the external application of magnets.

It appears that the magnet acts as a sedative or antispasmodic. Brimstone and camphor, applied externally to the body, have been found to act in the same manner. Hence we may derive another argument in favour of the existence of a magnetic fluid; for we can scarcely suppose that the magnet produces this effect by its merely attracting or repelling the particles of iron which are in the blood. But this seems to be put beyond all doubt by observing, that the magnet does not act upon the particles of blood until they have been calcined; and therefore can have no influence on the animal body merely by its attractive power.

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